

## COMMENTARIES

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# Fruits of Polyphony: A Commentary on a Multiperspective Analysis of Mathematical Discourse

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In a special issue on the role of artifacts in mathematics learning, it is interesting to call to mind the original meaning of the word *artifact*. The word is derived from the Latin words *ars* (meaning “art” or also “craftsmanship”) and *factum* (from *facere* = “to make”). So literally an artifact is “something made by craftsmanship.” In this sense it has been used for a long time in literature as an opposite to *natural* or *naturally given* (as one can still see in the use of the word *artificial*), but with the progress of social research there is a growing awareness of the fact that almost anything that we know of now in our culture is somehow artificial, that is, constructed by human beings.

According to Vygotsky (1982/1930), such artificial constructions serve as instruments for the mastery of processes in our surrounding nature and ourselves. In fact, Vygotsky often used the terms *instrumental* and *artificial* (*iskusstvennyj*) as synonyms. He furthermore emphasized that these artifacts often are related in complex ways; they are parts of cultural systems of activity, and as such they influence the course and structure of concrete human actions. During the accomplishment of instrumental actions, the structure of the instruments can be modified

gradually according to the requirements of the situation, which in turn may change the structure and course of human activities.

This issue of *The Journal of the Learning Sciences* concentrates on this very same process. The focus is described as being directed to “the ways in which symbolic tools and other artifacts enable, mediate, and shape mathematical thinking, while being themselves, at least to some extent, a product of these processes” (Sfard & McClain, 2002/this issue, p. 154). The approach chosen for getting a deeper understanding of this process is the analysis of two data sets taken from a classroom design experiment of Paul Cobb and his team. Two video clips of classroom discussions were analyzed by a number of specialists, who presented their in-depth analyses of the two sessions from their own perspectives. Given the complexity of the issue at hand, it is potentially very productive to ask different people, with varying theoretical interests, to analyze the two sessions. According to Bakhtin (1984), such “polyphony of different voices” is the basic condition for the growth of understanding. However, the mere presentation of the different analyses by themselves is not the best and most desirable outcome of such a multiperspective project. By itself, each analysis gives a highly interesting interpretation of the two target sessions, highlighting processes that are without doubt important for a deeper understanding of the development of mathematical thinking in the context of classrooms. But taken together, what can we learn from this polyphonic endeavor? What fruits can we harvest from this?

I believe there is a lot to be learned from this virtual discourse among these specialists in mathematics education, but the limited space forces me to focus on just a few issues. In the description of the main purpose of the issue we read no explicit restriction as to the kind of artifacts under study, so this description opens a very wide perspective on the development of mathematical thinking: All historically produced structures that possibly enable, mediate, and shape mathematical thinking can in principle be analyzed here, going from time tables, class composition, didactic strategies, and so forth to inscriptions, verbal concepts, computer programs, participant structures, and archives (Foucault, 1969). Nevertheless, most analysts focus on the minitool (or use of the minitool) and how it mediates the discourse and thinking processes of the participants. Given the target sessions, this restriction seems reasonable and absolutely worthwhile. Together, the various analyses demonstrate that mathematical problem solving in a group is not just an exchange or struggle of mathematical meanings, but is basically a permanent struggle of *mutual interpretations of meanings*: it is person A’s interpretation of what person B might be meaning, and vice versa, and the subsequent rejection, integration, or modification of this interpretation into new meanings A’ and B’, followed by exteriorizations of these digested meanings and so on. This discovery was well phrased by Kay McClain when she wrote, “In coming to understand the importance of taking account of their [the pupils’] actions and constructions, I began to work much harder at making sense of their activity instead of imposing my understanding.” So this teacher’s interpreta-

tions of the participants' contributions guided her interactions with the students and especially led to adjustments in her conjectures about the classroom community's learning trajectory. But, as McClain also pointed out, this is not simply a process of following the students' meanings. The teacher has responsibility for guarding the mathematical content as well. Basically, the teacher is permanently confronted with this tension (inherent in all teaching) between what she selectively perceives as the exigencies of the praxis involved and the meanings and interests of the pupils involved.

All contributors tried to explain in their own words parts of this complicated process of actually building learning trajectories in the classroom. From the different analyses it can be learned that there are many different dimensions to be taken into account here. Basically we can contend that the trajectory refers to each individual's development of his or her ability to participate in a collective practice. However, Paul Cobb's analysis shows us that this is not a one-way interiorization process going from the community to the members. In fact, the community also co-develops with its members on the basis of the exteriorizations of their meanings. His analysis illustrates that the classroom community's mathematical reasoning is interdependent on the participants' evolving chains of signification.

Together, these analyses show that the actual course of actions of any individual is always multiply determined. Human actions are always contextualized by a multilayered system of symbolic forces (e.g., Hicks, 1997). There is no space here to argue for a specific analytic system, but for the sake of the argument we can take the following levels as indicative of the dynamics in the evolution of actions (see also Saxe's contribution):

1. The structure of an activity that is constitutive of a particular practice: Several of the contributions of this special issue focus on constituents of an activity structure. In Cobb's contribution we can read how the object of an activity (the mathematical content) evolves and actually emerges out of the classroom conversation; every new conception of the content of the conversation partially co-determines the course of the conversations and the associated learning processes. Anna Sfard's analyses demonstrate, however, that the construction of a mathematical object essentially depends on the initial subjective conceptions (intimations) of the individual (based on previous experience) as well as on the consequences of their applications (implementation). But an activity is not only constituted by its object: Various other constituents are involved such as norms and motives (Cobb) and goals (Saxe). Activity is a multidimensional process that evolves microgenetically as one integrated whole (see also Saxe's explanation of microgenesis!). These evolving constituents of an activity system affect the course of a person's tool-mediated actions and as such inform the conjectures about possible learning trajectories.

2. Social interactions and discourse dynamics: In their actions and utterances, individuals also respond to events that originate from their social interactions and discourses. McClain produced interesting ideas about how a teacher may contribute to her pupils' development by taking part in the conversations of the pupils. We can learn from this that it is very important that all voices implicated in this polylogue are heard and taken into account (including the teacher's). But as Forman and Ansell convincingly pointed out in their analysis, this is not always or self-evidently a well-balanced process. Sometimes some voices are not taken into account or maybe even not heard. The evolution of individual learning processes (and a fortiori the associated evolution of the community's reasoning) depend on such selective mechanism in the community. Forman and Ansell's analysis calls for serious considerations of these selective mechanisms that determine the shifts in the participation structure of the community. We cannot preclude that tacit ideological assumptions and habits are working here. From Saxe's analysis we learn that social interactions may give rise to the emergence of new goals, but even here we might ask ourselves about the deeper foundations of the valorization of the specific goals.

3. Properties of the tools (artifacts): All authors somehow refer to the influence of the structure of the tools used in the design experiment. The minitool embodies mathematical structure and rules and constrains the pupils' actions and meaning construction. Sfard's analysis beautifully explains how the use of the tools (and consequently the students' actions and utterances) emerge out of an intricate interplay between intimations and implementations. It is plausible to assume that differently designed tools would evoke different associations. Some of the pupils' utterances can be understood only when we can see them as responses to the properties of the tools.

4. Individual mathematical understandings: Schliemann's analysis justifiably draws our attention to the individual students' perspectives, showing that the contributions of every participant in the classroom discourse can never be interpreted deterministically from the properties of the tools, the tendencies in the conversations, or the dynamics of the activity. Students assign meaning to different aspects of their environment, and this is often correlated to their ontogenetic level of development or to a priori assumptions originating from their personal learning histories. This was especially argued by Schliemann, but several authors in this issue clearly demonstrated the impact of personal qualities, for instance, in their analyses of the students' conception of the data as a self-contained number set or as a sample from a broader population. Such individual conceptions lead to different contextualizations and different ways of reasoning, like many authors showed. It might be that some of such conceptions do not originate from responses to the activity structure given or from responses to the features of the ongoing discourse or to the qualities of the presented tools, but they simply originate from the subjects' imaginative minds and personal history.

What can we learn from all these considerations? The ways in which symbolic tools enable, mediate, and shape mathematical thinking turns out to be a very complex process indeed, which includes a multiplicity of symbolic dimensions. An activity theoretical analysis (including an analysis of the communicative actions) offers a good perspective for guiding further research into the details of this multi-layered determination structure. It seems to me that further studies of the various selective mechanisms at the different levels are very important, both with regard to the individual and collective processes that underlie the emergence of new content and with regard to the shifts in the organization of the activity and its participant structure. In this regard, ideological issues like power, authority, and “truth regimes” are very important, but they were only noted in passing by some of the contributors. In a way, it is surprising that all contributors focused on the tool-mediated actions of the students, taking the tacit (often ideological) artifacts for granted. There is no way of ever getting a good understanding of symbolic tools without also analyzing the selective structures that settle tacitly in conversation regimes, attitudes, and habits. It is amazing to see to what extent these students and teachers already have developed a consent about what is relevant in a mathematics lesson, about what counts as “mathematics,” but where does it come from? This notion is not given from heaven but developed by education and reinforced time and time again in mathematics education. The ways in which tools can mediate and shape mathematical thinking probably is also constrained significantly by such “hidden” regimes. All articles in this issue can be read as evidence that points in that direction. That is at least one of the fruits of multiperspective discourse.

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