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Guest Editor's Introduction

Analyzing Tools: Perspectives on the Role of Designed Artifacts in Mathematics Learning

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> Concentrating the attention on one aspect makes it leap into the foreground and occupy the square, just as, with certain drawings, you have only to close your eyes and when you open them the perspective has changed. (Calvino, 1983, p. 7)

In the short story from which the preceding quotation is taken, Italo Calvino's thoughtful character, Mr. Palomar, is watching sea waves and is reflecting on the experience. In the constant quest for understanding, Mr. Palomar is never fully satisfied with what he already has managed to see. His looking is not a passive absorption of visual signals, nor is it considered as completed when a single wave has been described in finest details. Although successful in drawing the picture, Calvino's protagonist also realizes that "isolating one wave is not easy" (p. 3) and that "you cannot observe a wave without bearing in mind the complex features that concur in shaping it and the other, equally complex ones that the wave itself originates" (p. 4). In a series of strenuous trials, Mr. Palomar manages to create for himself a rich collection of pictures, each of which is quite meaningful in its own right, but none of which can be considered as telling the story of the waves unless taken together with

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all the others. "It would be disastrous if the images that Mr. Palomar has succeeded painstakingly in putting together were to shatter and be lost" (pp. 7–8), remarks Calvino.

Such shattering is only too likely to happen if the formidable task of collecting many different pictures is undertaken by a single person. The contributors to this special issue try to perform collectively what Mr. Palomar hoped to achieve on his own: They are all looking at the same phenomenon from a number of different perspectives. Rather than studying waves, they are observing a class of seventh graders making their first steps in analyzing statistical data. Rather than just describing the situation, the authors are applying different conceptual lenses to answer a set of well-defined questions about the process of learning. Still, their aim is not unlike Mr. Palomar's: In their collective effort, the writers wish to see as much as can be seen and with as much depth and precision as possible. Before elaborating on the ways in which they pursue this goal, let us say a few words about the phenomena under study and the questions asked.

The focus of all the analyses in this special issue is on the ways in which symbolic tools enable, mediate, and shape mathematical thinking, while being themselves, at least to some extent, a product of these processes. "Man differs from animals in that he can make and use tools", says Luria (1928, p. 493). Indeed, the use of specially designed artifacts is characteristic of any human activity, and of the activities of thinking and learning in particular. Most prominent in this latter category are semiotic tools such as language, specialized symbolic systems, and educational models. Although no student of cognition is likely to deny the paramount importance of symbolic tools in human thinking, the exact place ascribed to the semiotic artifacts would change according to one's underlying epistemological premises and to his or her perspective on cognition. As long as learning is conceptualized predominantly in terms of *acquiring knowledge*, or of a growth of mental structures, symbolic tools, even if seen as very important, would still be ascribed only a secondary role of means to a predefined end. Such tools may be believed to help in constructing knowledge and to have a considerable effect on the shape of mental schemes in which this knowledge is supposed to be stored; nevertheless, being regarded as not more than an optional, temporary vehicle for thought, they are likely to be of a greater interest to educators than to cognitive scientist. Indeed, when knowledge is conceived as an entity in its own right, subordinate to certain universal laws, any artifacts used in the process of knowledge acquisition must seem both replaceable and disposable: What should be learned by the student is believed to be attainable by dint of a whole range of symbolic tools and designed artifacts; once their job as construction aids is completed, the symbolic scaffoldings may leave the scene and need no longer be a part of the researcher's story. Within this well-developed, long-standing research tradition, symbols and other instructional artifacts are relegated to the role of external representations (Janvier, 1987; Kaput, 1991; for a

critical perspective see Meira, 1998). Their secondary status with respect to the knowledge itself is engraved in this name.

This picture changes rather dramatically the moment one chooses to replace the idea of learning-as-acquisition with the vision of learning as a "legitimate peripheral participation" (Lave & Wenger, 1991). Indeed, "participationists" view learning as initiation to a well-defined practice or discourse, such as, say, the discourse of the mathematical community or of computer scientists. Within the resulting framework, the focus of research shifts from states to actions and from a putative permanence of individual traits and possession to never-ceasing interaction and change. In consequence, those who adhere to this approach talk of *knowing* rather than *knowledge* (Cobb, 1995; Smith, 1995) and of *mathematizing* rather than *mathematics* (Sfard, 1997). Symbolic tools and other artifacts, being a part and parcel of action, become inextricable ingredients of the investigated phenomenon. Indeed, the view of symbolic artifacts as optional, disposable expressions of self-sustained mental constructs is incompatible with the idea of human learning as steadily improving coordination of an individual with a certain community (Greeno, 1997; cf. Wittgenstein's 1953 discussion of meaning and understanding and Vygotsky's [1987] critique of research that treats words and word meanings as separable entities). The developmental order implicated in the vision of tools as mere "helpers" or "enablers" of cognitive processes disappears, and rather than being optional avatars of independently existing mathematical ideas, the tools and their use are now thought of as a primary object of study (for a detailed argument on the untenability of claims on the precedence of concepts over symbols discussed from several perspectives, see Cobb, Yackel, & McClain, 2000).

Aware of the ontological commitments that come with words, those who choose the new perspective mark the conceptual transition by replacing the centrally important word *representation* with the word *inscription* (Latour, 1987). One reason that participationists eschew representation is the fact that this term always has been used in the double sense of the "external" and "internal" means for "keeping information." The association with "mental content" brings unwanted acquisitionist entailments (Roth & McGinn, 1998). Another, probably even more important, reason is the already noted dualism of the means of representing and the things represented implicit in the old terminology. The word *inscription*, preferred by the participationists, is supposed to be free from such connotations. Rather than being thought of as a way to record an independently existing "information," the term *inscription* is to be understood as referring to graphical displays created and used for the sake of communication. Following Forman and Ansell (2002/this issue), we may quote Lehrer, Schauble, and Petrosino (2001) and say that inscription is

> a term that we use to include drawings, maps, diagrams, text, recordings from instruments, mathematical formalisms of various kinds, and even physical models, serve to preserve, compose, and make public parts of the world so that they can be subjected to

> argument, they can be progressively built up and elaborated upon, and their history can be captured and preserved. Inscriptions do not merely copy the world; they select and enhance aspects of it, making visible new features and relations that cannot be seen by observing the objects and events themselves. For example, a road map selects and enhances aspects such as distance relationships and scale that are not visible to an individual at 'ground level,' while leaving out other features that are not important to the purposes of the inscription—such as trees, power lines, and buildings. (p. 259)

Thus, when in one of the episodes analyzed in this volume the authors speak extensively about problem solvers' inscriptions, they do not regard the inscriptions as mere optional expressions (representations) of children's solutions, but rather as the solutions themselves.

Inscriptions are but one of the many types of tools that mediate human communication. Other symbolic mediators massively present in today's classrooms are spoken and written language and interactive computer displays. For those who subscribe to the participationist framework, one of its most appealing aspects is that by presenting learning as inseparable from symbolic tools, it makes it impossible to put cognition apart from social, cultural, and historical factors. Indeed, attention to artifacts imposes the vision of cognition as a culturally shaped phenomenon. This inseparability is what puts tools, their construction, and their use at the heart of the current sociocultural treatments of cognitive development (cf. Cole, 1996; Scribner & Cole, 1981; Vygotsky, 1978, 1987) and of the ongoing debates on emergent classroom practices (Ball, 1993; Cobb et al., 2000; Lampert, 1985; Roth & McGinn, 1998). Rejection of the essentialist account of symbolizing inevitably leads to the conclusion that almost anything that goes into our conceptualization of the world is "accomplished through public discursive practices in which objects, images, diagrams, talk, the body, standards, encompassing activities, and so on play a central role" (Goodwin, 2000, p. 1).

Although part and parcel of discursive processes, symbolic tools are, in themselves, discursive constructs. The way new symbolic mediators, supposed to help in solving particular problems, come into being in complex discursive processes is the leading theme of the analyses included in this volume. The reflexive nature of symbolic artifacts—the fact that they are products of the very same communication that they are supposed to make possible in the first place—implies the inherent circularity of the construction processes. The authors whose analyses of this process may be found in this volume seem well aware of the paradoxical nature of the phenomenon and of the resulting difficulty of the task they have undertaken. The challenge they face brings to mind the famous learning paradox (Bereiter, 1985) and the celebrated hermeneutic circle (Gadamer, 1975), both of which, just like the circularity noted here, make the processes of learning and understanding into an inexhaustible source of puzzlement and an everlasting object of study.

In an attempt to meet the challenge, each contributing author has developed her or his own perspective and methodology. The approaches offered by each author differ in their foci, questions asked, organizing principles, and methods of analysis. Thus, although all the authors share the conviction about the centrality of cultural and social factors, they differ in their specific interests and emphases. At the same time, they are acutely aware of the old truth that fueled Mr. Palomar's investigations: One perspective, be it as revealing and rich in possibilities as it may, can never be enough. Thus, the purpose of this special issue is to present and contrast the differing approaches while trying to show that they are complementary rather than mutually exclusive.

The first step of all the contributors is to present their conceptual frameworks and to describe the main tenets of the theories that guide their research. The fact that all the authors are unified in their strong commitment to the idea of sociocultural embeddedness of cognition does not mean that they speak in one voice. The approaches that can be found in this volume cover a wide spectrum of sociocultural theorizing. Although the metaphor of learning-as-becoming-a-participant clearly prevails in most of the contributions, none of the writers rejects the acquisition metaphor altogether. Similarly, both extremes of the individual–collective axis can be found among the diverse units of analysis chosen by the different authors. What Mr. Palomar attained single-handedly by studying, first, a single wave and then the ways in which different waves interact with each other, our contributors achieve by splitting the task between themselves. Thus, for example, whereas Analúcia Schliemann points out that the individual student's perspective should never disappear from the researcher's sight, Paul Cobb argues for the advantages of the encompassing vision of collective classroom practices. The reader may wish to keep Cobb's definition of practice in mind while reading Ellice Forman and Ellen Ansell's insights about similarities between children's classroom activities and the patterns of action to be found in scientific communities, while following Kay McClain or Anna Sfard in their respective accounts of the evolving forms and norms of classroom communication, or while studying Geoff Saxe's sociogenetic analysis of mathematical learning. It must be stressed, however, that just like Mr. Palomar, none of the contributions uses just one unit of analysis. For Saxe, multidimensional treatment involving three different levels of analysis—microgenetic, sociogenetic, and ontogenetic—is a matter of a theoretical principle. The general message of this and similar mixtures of perspectives to be found in other contributions is that the attention to cultural and social factors does not preclude the study of an individual. Indeed, all the authors seem to agree that there is no contradiction between focusing on the individual and the claim on the social nature of cognition. After all, if learning is inherently social, it remains so even when performed by a single person.

To enable a sharp focus and a clear vision of similarities, differences, and complementary aspects of the various perspectives, all the authors illustrate their theo-

retical claims with analyses of the same video-recorded classroom episodes. The episodes come from an experiment conducted by Paul Cobb, Kay McClain, and Koeno Gravemeijer in which a class of seventh-graders is being introduced to statistics (Cobb, 1999; McClain & Cobb, in press; McClain, Cobb, & Gravemeijer, 2000). The episodes feature students solving statistical problems and using computer-based tools for exploratory data analysis. One may ask what makes the brief classroom events worth so much attention. While working on this project we heard people wondering how many useful insights might possibly be found in the brief isolated fragments of seemingly uneventful school conversations. On the face of it, only a very limited amount of insight seems possible. We hope that the analyses gathered in this volume prove this belief wrong. Joining forces, the authors in this issue are making the familiar unfamiliar. They show that whenever we manage to rise above everyday unreflective understandings, we find questions whose answers are far from obvious.

The articles by Cobb and by McClain highlight participants' outlook, coming from people who, in the original study, served not just as researchers but also as teachers. The remaining four articles offer external observers' perspectives. Although all the articles deal with just two brief learning events taken from different stages of the teaching experiment, the insiders' analyses of those isolated "waves" are informed by their extensive knowledge of the waves that preceded and that followed the ones under study. This is not the case for the four teams of nonparticipants. he differences in the degree of familiarity with the general context and in the theo - retical approaches result in interpretations that differ in their status. In any case, analyses based on just two brief episodes should count as exercises in applying certain analytic methods rather than as attempts to say anything general about the learning that took place in the project classroom.

Different ways of interpreting, like Mr. Palomar's different ways of looking, highlight many diverse aspects of the two classroom events. Cobb conceptualizes the mathematical learning in terms of developing norms of classroom community. His analysis, focusing on the collective shaping of what is called here *normative ways of reasoning with symbolic tools,* reveals chains of signification that emerge while students attempt to find helpful ways of interpreting graphic displays of data. Kay McClain tells her story as a teacher and, while doing so, reflects on her own learning that took place in the course of the teaching experiment. Her account makes it abundantly clear how difficult it is for a teacher, who is well aware of the designer's intentions inscribed into a symbolic tool, to put aside her own understanding to communicate with students on their own terms. Whereas McClain focuses mainly on teacher–student interaction, Forman and Ansell take a close look at the way in which symbolic tools shape *argumentative positions* of the students. Their conclusion is that students' ways with inscriptions are comparable to those of professional scientists. The pivotal idea of Saxe's account is that of *emerging mathematical goals*, and his focus is on the way children's use of artifacts enables

and constrains particular kinds of goals. His multilevel analysis aims at illuminating the interplay of cultural and developmental factors in children's mathematical learning. Schliemann's declared intention is to show that understanding actions of the individual is as crucial to understanding of the social and cultural dynamics of learning as the study of a single wave is to the study of the dynamics of overall wave movement. In her account, she tries to identify the sources of discursive decisions made by individual interlocutors. Finally, Sfard uses what she calls *communicational approach* to uncover the ways in which discursive uses of computer minitools are interactively constructed by the students. Her analysis shows that the circular interdependence of tool creation and tool use may be overcome by a subtle interplay of intimations and implementations, that is, by a dialectic adjustment of old discursive habits to new contexts.

Organizing the six articles around a single set of data presented the authors and editors with special technical difficulty. The challenge was to find a subtle balance between brevity and comprehensibility, that is, to organize the different texts in such a way as to avoid tiresome repetitions and, at the same time, to ensure the clarity of the articles. Our solution was to separate the materials that are common to all the articles from the articles themselves. As a result, the detailed description of the project, the tools, and the episodes, as well as the transcripts of the classroom conversations and inscriptions drawn by the children, may be found in the following article by Kay McClain, which we refer to as the Appendix.[1] This information should be regarded as an integral part of each of following six articles.

This organization of the content may be quite unusual, but it also can be somewhat problematic because it imposes a certain nonlinearity of reading, and such nonlinearity conflicts with the linear nature of text. This kind of problem is best solved by replacing the ordinary text with hypertext. Indeed, appended to this volume is a CD-ROM that contains hypertext versions of all the articles together with video clips presenting the classroom episodes. The episodes are subtitled, and the hypertext enables smooth transitions from the transcript lines quoted in the articles to the corresponding segments of video clips. This additional form in which the material is offered to the reader is one feature that makes this issue really special.[2]

The challenges of this special project would not have been met if not for the help of many people. Our thanks go, first and foremost, to Brian MacWhinney, who volunteered all his inventiveness, much of his time, and a lot of work and good will into reincarnating Geoff Saxe's modest idea of a volume accompanied by video clips into

[1]In the Appendix, participants' statements are numbered sequentially. In the articles that follow the Appendix, these statements are referred to by their number (in brackets).

[2]The reader who prefers to use the paper copy of the special issue may take advantage of the CD version to print a copy of the Appendix from the accompanying CD-ROM. The printout of the transcripts and figures then may serve as a handy reference when the articles and commentaries are being read.

a smoothly working, stand-alone, electronic edition of the special issue. We are also deeply grateful to the many colleagues who agreed to take part in the rather demanding and quite unusual review process. We thus wish to thank Phil Bell, David Bloome, Jere Confrey, Jim Gee, Shelley Goldman, Patrick Gonzales, Rogers Hall, Jey Lemke, Sten Ludvigsen, Doug Macbeth, Judit Moschkovich, Ann Ryu, Emily van Zee, Barbara Wasson, and Jim Wertsch, all of whom contributed to this issue by generously sharing their knowledge and by providing the authors with insightful suggestions. Special acknowledgment goes to our four commentators—Bert van Oers, Magdalene Lampert, Neil Mercer, and Douglas Macbeth—who, undeterred by the tight deadline, did a splendid job in pulling together the many threads spread along these pages. Last, we wish to express our gratitude to *JLS* special issues editor Tim Koschmann and his assistant, Sandy Birdsell, for letting us capitalize on their experience and organizational talents, and for guiding us, ever so gently, through the mazes of the editorial work.

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