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To cite this article: Carla C. van de Sande & James G. Greeno (2012) Achieving Alignment of Perspectival Framings in Problem-Solving Discourse, Journal of the Learning Sciences, 21:1, 1-44, DOI: [10.1080/10508406.2011.639000](https://doi.org/10.1080/10508406.2011.639000)

To link to this article: <https://doi.org/10.1080/10508406.2011.639000>



Published online: 10 Feb 2012.



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Achieving Alignment of Perspectival Framings in Problem-Solving Discourse

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We use a concept of framing to explain 3 cases in which participants initially lacked mutual understanding but then achieved significant mutual understanding. The cases were all consistent with a pattern of *positional framing* that includes a human participant who is inquiring, which we call a *listener*, and a *source*, which may be another human participant or a non-human system. Participants with these different positionings have different perspectives that shape the interpretations and functions of their contributions. We hypothesize that achievement of mutual understanding also depends on participants having *epistemological* and *conceptual framings* that are sufficiently aligned. We distinguish between cases in which the participants achieved alignment of conceptual framing by adopting a schema that was in their common ground, cases in which the participants co-constructed a coherent framing without the benefit of a prefabricated schema (by constraint satisfaction, we hypothesize), and cases in which mutual understanding involved a refinement that did not require reframing. We argue that our hypotheses qualify as explanatory mechanisms, and we explicate this in an appendix.

Our general goal is to contribute to the growing body of scientific studies that include concepts and methods for understanding cognitive structures and processes integrated with concepts and methods for understanding interaction that people have with each other and with other material and informational systems. [Supplementary materials are available for this article. Go to the publisher's online edition of the *Journal of the Learning Sciences* for the following free supplemental resource:

“A Mechanistic Analysis of Discourse,” which presents an analysis of an episode from Case #1 of this article as a more detailed example of the kind of mechanism we have hypothesized in our more general accounts of interaction presented in the article.]

BACKGROUND

The integrative idea that we propose is an extended version of framing. Generally speaking, *framing* refers to the way in which participants understand the activity they are engaged in, sometimes characterized as an answer to the question “What is it that’s going on here?” (Goffman, 1974/1986).

In our use of the concept of framing we focus on ways in which activities are organized. We distinguish three interrelated aspects of framing, which we call *positional*, *epistemological*, and *conceptual framing* (or more simply *positional* and *cognitive framing*).

By *positional framing* we refer to the way in which participants understand themselves and one another to be related to one another in the interaction, especially regarding the kinds of contributions each of them is entitled, expected, and perhaps obligated to make in the group’s activity. Our use of this term refers to aspects of interaction that were considered by Bateson (1955/1972) when he introduced the term *frame* and in subsequent discussions by sociologists and sociolinguists such as Goffman (1974/1986), Gumperz (1982), and Tannen (1993). In our view, several terms in the literature, including *participant structures* (Philips, 1972), *participant frameworks* (Goodwin, 1990; O’Connor & Michaels, 1996), *positioning* (Harré & van Langenhove, 1999), and *footing* (Goffman, 1981), have significant similarities in meaning, and they coincide with aspects of our use of the term *positional framing*.

The concept of cognitive framing¹ that we use includes epistemological framing, an idea that is being developed by Hammer and his associates (Hammer, Elby, Scherr, & Redish, 2005; Scherr & Hammer, 2009). *Epistemological framing* refers to participants’ understanding of kinds of knowledge that are relevant for use in their activity and the kinds of knowledge, understanding, and information they need to construct to succeed in their activity (e.g., what kind of information would count as a solution to the problem they are working on).

We call another aspect of cognitive framing *conceptual framing*. This term refers to the way or ways in which participants organize information in the situation they are discussing and the problem they are working on. In our use of the term, *conceptual framing* refers to relations between different aspects of information in the situation, including those that are attended to and those that are not attended to, and those that are in the foreground and those that are in

¹We agree with Kirsh (2009) that analyses of problem solving in cognitive science have neglected important aspects of cognitive framing, and we hope that our analyses contribute toward remedying that.

the background of attention. We consider conceptual framing as a version of the problem of cognitive organization studied classically by Gestalt psychologists (e.g., Duncker, 1935/1945; Katona, 1940/1967; Luchins, 1942; Wertheimer, 1945/1959). We also consider conceptual framing as a generalization of the concept of schema. If a known schema is applied it determines an organization of information, hence a conceptual framing. Sometimes a schema is not known or accessed by participants, and then a conceptual framing can be assembled in a process that we hypothesize to involve constraint satisfaction.²

We also draw on an idea that cognition is fundamentally perspectival, that is, that cognition always has a point of view, which we include in our accounts of cognition in activity. Rommetveit (1974, 1998) developed arguments that alignment of perspectives is a necessary condition for understanding the intended meanings of sentences and messages,³ and MacWhinney (2005) argued that syntactic features function to signal shifts in perspective, supporting comprehension of sentences.

Our analyses of interaction in terms of framing adopt activity systems as our primary analytical focus; that is, we take what Greeno and others (e.g., Engle, 2006; Greeno, 2011; Greeno, Collins, & Resnick, 1996; Putnam & Borke, 2000) have called a *situative theoretical perspective*. We consider groups of individuals interacting with each other and with other material and informational resources as cognitive systems.⁴ That is, like Hutchins (1991, 1995a, 1995b), Weick and Roberts (1993), and others we consider cognitive processes as being distributed

²The concept of *constraint* that we use is very general, developed in situation theory (Barwise & Perry, 1983). Attunements to constraints include cognizance of requirements (e.g., in Case 2, discussed later, it was relevant that solving an algebra problem with two unknown variables requires two equations) and knowing regularities in the environment (e.g., in Case 3 it was relevant that the shape and direction of one arrow in the display was programmed to coincide with a change in another arrow, also in the display).

³An example by Rommetveit (1980) illustrates framing with significant positional and cognitive aspects: Mr. Smith is mowing the lawn at his and Mrs. Smith's house on a Saturday morning. Rommetveit (1980) discussed two hypothetical phone calls, both of which ask Mrs. Smith "Is he working?" Although the two callers ask questions using the same words, Mrs. Smith would answer them with different responses. If the caller was Mr. Smith's frequent golfing partner, Mrs. Smith would correctly frame the question as asking whether Mr. Smith was working at his office and answer "No, he's mowing the lawn." But if the caller was a nosy neighbor known to disapprove of late-night partying, Mrs. Smith would correctly frame the question as asking whether Mr. Smith was working rather than still sleeping and answer "Yes, he's mowing the lawn." To answer "Is he working?" correctly Mrs. Smith would adopt a conceptual framing that is determined by the caller's positional relationship with Mr. Smith, which, epistemologically speaking, makes different information about Mr. Smith (no, he is not working at his office; or yes, he is awake and active) relevant to answering the question informatively.

⁴As Goldstone and Gureckis (2009) put it, "Indeed, one might go so far as to say that groups of people themselves can be interpreted as information processing systems" (p. 415). We also include other material and informational systems that are in the groups' environments, along with groups of people, as components of the systems that process and construct information.

across individual people and other resources, and we follow Engeström (2001) in referring to collections of interacting individual people and resources as *activity systems*.

Furthermore, we apply this situative perspective in analyses of conversational discourse. We treat conversation as a cognitive system, that is, a system that processes and constructs information. Following Clark (1996), Brennan and Hanna (2009), and others we conceptualize conversational communication as joint action. Like Clark and Schaefer (1989) and others, we identify exchanges that add information to a conversational common ground as contributions that are accomplished when participants give each other evidence that they have achieved mutual understanding.

ACHIEVING ALIGNED FRAMINGS

We integrate these ideas of framing, cognition distributed in activity, and conversation as joint action by considering the alignment of framing as a condition for achieving mutual understanding, which is constructed through talk in interaction. We propose a scheme for analyzing episodes of interaction in which participants initially lack mutual understanding of something that is significant in their activity, this lack of understanding is recognized, the participants jointly engage in an effort to reach mutual understanding, and the participants achieve mutual understanding that they accept as sufficient for their purpose.

The process that we hypothesize has three general phases. First, the participants detect that at least one of them is not understanding some important information. The information not being understood can be from another person or other persons, or it can be from some other source, such as a written text or an interactive computer program. Our hypotheses are relevant when the lack of understanding involves a significant level of meaning rather than someone simply not hearing or mishearing what is said or a listener interpreting an ambiguous statement differently than the speaker. We consider examples in which participants detect a lack of understanding that they take to require significant inquiry at the level of meaning, which we hypothesize as being due to a misalignment of their framings.

Second, the participants jointly engage in an effort to achieve mutual understanding sufficient for their main purpose and construct information in their common ground. This involves their knowing and adopting a practice that we previously called *constructive listening* (Greeno & van de Sande, 2007), which we believe refers to the aspect of practice that Engle and Conant (2002) called *problematizing*, that Mercer (1995) called *exploratory talk*, and that Weick and Roberts (1993) called *heedful interrelating*, in which participants consider differences in their understandings and try to construct a relevant mutual understanding in their common ground.

In this ensuing interaction an aspect of positional framing is required for the mechanism to operate. Someone or something has to be positioned as a source of information for constructing an understanding that at least one of the participants lacks, and another participant (or other participants) need(s) to engage in an effort to understand the information that can be constructed by interpreting the source in a way that results in mutual understanding. We refer to the participant(s) whose positioning involves making this effort as the *listener*, and the individual(s) or other system that provides information as the *source*. These aspects of positioning, like many others, can shift among participants during an interaction. The listener may or may not initiate exchanges by asking questions; in either case, the listener is engaged actively in inquiry.

As a result of this positioning participants have different perspectives on information that matters for accomplishing their joint task. In the listener's perspective there are inconsistencies, gaps, or unsatisfied constraints that give rise to questions that the listener hopes to answer by asking or observing the source carefully. In the perspective of the source (if the source is a person or persons, assuming that he or she cooperates with the questioner) it is thought that information needed to answer the listener's questions may be accessible to the source. Of course, along with this difference between perspectives, which is due to different positionings, the participants' general perspectives on the task (especially their epistemological perspectives) also need to be generally aligned to support their communication.

We distinguish between different cases of participants working to achieve mutual understanding that differ in complexity. In relatively simple cases mutual understanding can be achieved if the participants activate a schema that they both (all) know, which supports their organizing information provided by the source, that is, their conceptual framing, with sufficient alignment for mutual understanding that serves their purposes.

In more complicated cases a prefabricated schema sufficient for mutual understanding is not known (or perhaps not activated) by the participants. Then we hypothesize that the participant(s) who is (or are) positioned as the listener need(s) to construct a framing that is sufficiently aligned with the framing that the source has. A conceptual framing involves the way in which information is organized so that information provided by the source can be incorporated readily into the common ground.

We hypothesize that when participants construct a shared conceptual framing (i.e., when they do not achieve this by activating a single shared schema) the knowledge that supports this process of inquiry is attunement to constraints. The listener(s) attend(s) to the source to determine whether, and, if so, how, the information presented by the source satisfies constraints that are taken as relevant. Constructing mutual understanding in this kind of interaction is a joint effort, with both the listener and the source (when both are human) understanding that they need to construct explanations that satisfy general constraints.

The third phase of this process of inquiry is its termination, which occurs when the participants have constructed a framing that they take as satisfying the condition of sufficient mutual understanding. This can be indicated by their succeeding in including in their common ground information that previously they were unable to understand mutually, to their satisfaction.

We believe that this model of a process of reaching mutual understanding is an example of mechanistic explanation as it has been discussed by Bechtel (2007), Machamer, Darden, and Craver (2000), and others. We discuss this in some detail in an appendix available in the publisher's online edition of *Journal of the Learning Sciences*.

DATA SOURCES

We present analyses of three cases of interaction. We selected each of these because when the interactions began participants did not understand each other in some way, or they did not understand something significant about the situation, but they subsequently achieved significant mutual understanding. These cases represent different patterns of misalignments of cognitive framings that participants needed to overcome and of positional framings that they adopted in the activity in which they achieved mutual understanding.

Two of the cases are taken from video records of two eighth-grade algebra classes made over the course of a school year, with records of every class meeting for the first 5 weeks of the school year and records of approximately one meeting per week of each class for the rest of the year. In each recorded class one camera followed the teacher, who wore a wireless microphone. Another camera focused on one group of three or four students who worked together on tasks specified by the teacher. Details of this setting and data collection were presented in Gresalfi (2004, 2009). The other case that we present is a reanalysis of an episode of interaction reported by Roschelle (1992). In Roschelle's example two high school students worked together with an interactive computer program to solve challenges that involved setting parameters of a simulation system so that the motion of a pictured particle matched the pictured motion of a ball. In our reanalysis we rely on excerpts of transcriptions and descriptions that Roschelle included in his report.

CASE 1: FRAMING AND REPRESENTING PATTERNS OF DIAGRAMS AND NUMBERS

Our first case, from one of the algebra classes, is an episode that occurred less than 2 months into the school year, in October. The class activity was homework check, a standard activity type, conducted in groups of three or four students. Students were expected to compare their answers and resolve differences. The

teacher, whom we call Ms. Sanchez, emphasized practices of collaborative learning, including reminding student groups that they were expected to reach mutual understanding in their interactions. The homework task that the students were checking is shown in Figure 1.

Our analysis focuses on one of the student groups, made up of four students, whom we call Dani, Gillian, Jacinda, and Karin. The case comprises five episodes. The processes that we hypothesize to account for the participants reaching mutual understanding are quite straightforward compared with the other cases we analyze. Regarding positional framing, in each episode of Case 1 the student positioned as the source was also positioned as taking the initiative in the conversation, and in the successful episodes the listener was positioned to attend to and try to understand an explanation provided by the source. Regarding cognitive framing, we hypothesize that schemata known in common by participants are sufficient to explain their reaching most of the mutual understandings that they achieved.

Episode 1a: Understanding a Numerical Answer

Dani, Jacinda, and Gillian had written different answers for the first question and needed to resolve the difference. Their accomplishment in about $1\frac{1}{4}$ min of interaction was to reach agreement on an answer that they wrote on their homework sheets, with Dani and Gillian replacing the answers they had written previously. During this episode Dani, positioned as the source, presented a solution by explicitly carrying out an iterative procedure of adding the same number (2), keeping track of the step number corresponding to each of her results, and stopping when she reached step number 10, as specified in the question. Gillian was positioned as the listener. Gillian accepted Dani's result explicitly, and we infer that Gillian also accepted the process that Dani used. Given this inference it is reasonable to infer that the information Dani and Gillian constructed in their common ground included (a) the answer that Dani derived (21) and (b) its justification of having been the result of a process of iterative addition. Although it is possible that Dani and Gillian's common ground included a schema for performing and understanding this process, we make a different inference, that they both were attuned to a constraint that each addition corresponded to the difference between two successive items and that the result of adding that difference the correct number of times gave the correct result. By conceptualizing their understanding as constraint satisfaction we hold open the possibility that their understandings were partially misaligned. To explain Episode 1d we hypothesize that Dani's understanding was framed with numbers and operations in the foreground, with properties of diagrams backgrounded or absent, but that Gillian's understanding was framed with diagrams and differences between diagrams in the foreground, with numerical values considered as properties of the diagrams. This difference might not have been evident to Dani and Gillian in their interaction in Episode 1a. In any case

Exploring Patterns

For each pattern, sketch steps 4 and 5. Then predict how many units would be in step 10. Describe the pattern.

Function 1. Count the toothpicks.



4

5

How many would be in step 10? _____
 How is the pattern changing or growing? _____

Function 2. Count the toothpicks.

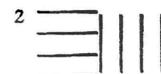


4

5

How many would be in step 10? _____
 How is the pattern changing or growing? _____

Function 3. Count the toothpicks.



5

How many would be in step 10? _____
 How is the pattern changing or growing? _____

FIGURE 1 Exploring Patterns assignment, page 1 of worksheet. From *The Pattern and Function Connection*, by Brad Fulton and Bill Lombard. Used with permission from Key Curriculum Press, www.keypress.com.

TABLE 1
Summary of Hypothesized Framings in Episode 1a

<i>Framing</i>	<i>Before Reframing</i>	<i>After Reframing</i>
Components in foreground	Numbers written as answers	An answer, 21, and a process of incrementally adding numbers (Dani) or a number of segments that denote toothpicks (Gillian)
Relations between components	Different answers written by Dani, Gillian, and Jacinda	The answer 21 is the result of the incremental adding process

Dani's procedure gave the correct answer, whether it was interpreted as referring to additions of numbers or to modifications of diagrams by adding line segments.

Table 1 is a summary of this interpretation.

The group, mainly Dani and Gillian, had the following conversation:

- 4:03 1. Gillian (G): 'kay guys, so, let's start with function one.
 2. Dani (D): Function uno. For the, um, how many'd be in step ten. What'd you get? I got nineteen.
 3. Karin (K): (*filling in answers on her homework sheet*)
 4. G: I got twenty-two.
 5. Jacinda (J): I got twenty-one.
- 4:17 6. D: Let's seee. Nineteen, twenty-one, twenty-two—shoot.
 7. G: (*tapped her homework sheet with a pencil, apparently counting the toothpicks shown in the fifth diagram*)
 8. J: What are you guys talking about?
 9. D: Function one.
- 4:28 10. J: Oh, that? I got twenty-one on that.
 11. D: Okay, it's one, two—you add two more—you keep adding two each time, so then—one, two—one, two
 12. G: ten, eleven, twelve, thirteen, fourteen, fifteen—wait, what? no
 13. D: (*tapping, apparently counting toothpicks on her homework sheet*) one, two, three, four, five six, seven, eight, nine, ten, eleven,
 4:48 and then, that'd be eleven for five. Six it would be thirteen. Seven it would be fifteen.
 14. K: You know, if you kept your last night's homework you would have had no problem—your last night's homework.
- 5:04 15. D: Eight would be seventeen. Nine would be nineteen, Yeah, and ten'll be twenty-one.
 16. G: Twenty-one?
 17. D: Yeah.
- 5:13 18. G: I must have been off by one.
 19. D: (*erased something on her homework sheet*) Yeah, I think I counted it <>⁵

⁵ <> indicates unintelligible talk.

Interpreting this interaction in terms of our hypotheses, we argue that the group detected a lack of mutual understanding when they reported the answers they had written and found that they differed. Dani took a source's position, which the others accepted, and was positioned as leader of the process of finding the correct answer. Dani explicitly stated a process of "adding two each time" and computed the answer by announcing the sequence of numbers that resulted from that operation corresponding to the sixth, seventh, eighth, ninth, and tenth items in the sequence.

Gillian, positioned as the listener, attended to Dani's exhibition of the process of successive additions of 2 and accepted the result as a correction of the number she had. Dani and Gillian provided explicit evidence that they had reached mutual understanding when they accepted the numerical answer 21.

The evidence that we have about Gillian and Dani's mutual understanding (which is approximately the same as the evidence they gave each other) is not sufficient to support a claim that their understandings were entirely the same. Partly because of evidence from Episode 1d we believe that it is probable that their understandings differed somewhat, although they did have mutual understanding sufficient for their main purpose, which we infer was to determine the correct answer to write on their homework sheets. We hypothesize that when Dani said "You keep adding two each time" there were two possible interpretations that were consistent with each other at the level of specifying a process of iterative addition to find the number of toothpicks in Diagram 10. But the two interpretations that we hypothesize differ in their referential meanings. The students had drawings of the first five diagrams and determined that their numbers of toothpicks were as follows: (1) 3, (2) 5, (3) 7, (4) 9, and (5) 11. One possible meaning of "You keep adding two each time" is a numerical pattern, which may be detached from reference to the diagrams. Another possible meaning of "You keep adding two each time" is a repeated action of adding two segments to the previous diagram, so that "two" is interpreted as referring to two toothpicks (represented diagrammatically as two line segments). With the former interpretation the meanings of "3," "5," . . . "21" are just the numbers, which increase by 2 with each increment. With the latter interpretation the meaning of "3" is three toothpicks, the meaning of "5" is five toothpicks, and so on; that is, the numerals represent a numerical property of the respective diagrams, the number of toothpicks they represent, and that numerical quantity increases by two toothpicks with each increment. We hypothesize that Dani and Gillian's common ground included attunement to a constraint that repeated addition of a fixed number, or numerical quantity, provided the correct sequence of numerical values. Dani generated the sequence of numerals by incrementing each number (or quantity) by a fixed numerical amount, keeping track of the number of increments, and keeping a running total of the results of the addition until the required number of increments had been made, and Gillian accepted Dani's presentation. However,

based on evidence in Episode 1d we hypothesize that Dani's understanding satisfied the constraint by adding numbers and that Gillian's understanding satisfied the constraint by adding numbers of toothpicks. Either meaning of the numerals is consistent with the overt actions that Dani performed to determine the numerical answer.

In terms of cognitive framings we hypothesize that Dani, Gillian, and Jacinda (and probably Karin as well) all shared an epistemological framing in which the paramount aim was to determine the correct answer to the homework question. Dani and Gillian also shared an epistemological framing in which the process of incremental addition was a valid method for determining the answer and an adequate justification for accepting the result.

The two differing interpretations we hypothesize involve conceptual framings. In the numbers-only interpretation, numbers are in the foreground of the framing and the diagrams are omitted. In the numerical-quantities interpretation, properties of the diagrams remain foregrounded in the framing, with numbers having a secondary status. This distinction is not required by the data of Episode 1a, although we later argue that it explains the data of Episode 1d. The aspect of our claim that is supported here is that the mutual understanding achieved by Dani and Gillian included (a) that 21 was the correct answer and (b) that the process of repeated addition presented by Dani provided a justification of that answer.

Episode 1b: An Interruption Prevents Reaching Alignment

The next excerpt of transcription includes a brief exchange in which Jacinda proposed an easier, but incorrect, alternative way to obtain the answer to the first question. A lack of mutual understanding was detected by Gillian, who responded to Jacinda with an explanation of the flaw in her proposal. Jacinda did not respond positively to Gillian's disagreement.

Jacinda presented her proposal after Gillian presented her answer to the next question on the worksheet ("How is the pattern changing or growing?"), and Dani responded to Gillian's proposed answer, ignoring Jacinda's presentation.

20. G: I I did three plus n minus one, times two.
- 5:20 21. J: (*looking down at her homework*) Well, if you would have just thought of five . . . and <have> the next one, eleven plus eleven (*looked and gestured at G*)
22. D: Wait, multiply the step number by . . . then add one.
23. G: (*looking down at her own homework*) That wouldn't work, though, because you start with one two three. And then you go one two, one two, one two (*looking at J*)
- 5:33 24. D: Wait (*looking at G, pointed toward G with her pencil*) Take the step number, multiply by two, and add one.

25. J: (*looking at G, pointed toward G with her pencil*) It worked with <xx xx xx> (*turned back to her homework, wrote on her homework sheet*)26.

G: (*looking at D*) Take the what?

Jacinda's proposal framed the problem as a simple proportion.⁶ We do not understand the discrepancy between her proposal (which would make the answer 22) and the correct answer, 21, which had been the answer given by Jacinda and accepted by the group after Dani's demonstration of iterative adding. Gillian responded by stating the property of the iterative addition process that made Jacinda's proposal flawed, that is, "you start with . . . three" (not two) so the function was not a simple proportion. We hypothesize that Gillian's response to Jacinda depended on a schema of proportionality that included a requirement that every increment, including the first one, have the same value; this is equivalent to knowing that the intercept of the function would have to be zero, but Gillian focused on a requirement that $f(1)$ would have to equal the slope of the function for the function to be simply proportional.

For Gillian's effort to reach mutual understanding with Jacinda to be effective, Jacinda could have taken up a position as a listener, making an effort to align her framing with Gillian's. Instead, Jacinda persisted, apparently saying that her proposed framing was consistent with another problem. At this point Gillian could have responded in a further effort as a source to achieve mutual understanding with Jacinda. Instead, Gillian responded as a listener to Dani, who had repeated her response to Gillian's earlier proposal of a formula to answer the second question. We interpret this as an interruption of the mechanism of reaching mutual understanding, so that in this instance it did not reach the terminating state. Indeed, Jacinda did not engage in the group activity after this point and instead worked by herself for the remainder of the time. We hypothesize that this kind of withdrawal may be typical of situations in which a participant trying to contribute is not positioned as a source.

Episode 1c: Achieving Mutual Understanding of Dani's Formula

In this episode Gillian and Dani completed their work on Function 1 and Gillian accepted Dani's formula to calculate the numerical answer to another question. In our interpretation, this episode is analogous to Episode 1a. Gillian and Dani had written different answers to the question about the pattern. Gillian had written $3 + (n - 1) \times 2$ (or something equivalent; see turn 20), and Dani had written $2n + 1$ (or something equivalent). As she had in Episode 1a Dani took a position as a source to explain to Gillian and persuade her that Dani's formula was a correct answer. She began this in turns 22 and 24 and continued in the subsequent dialogue

⁶We infer that Jacinda was assuming that if $f(5) = 11$, then $f(10) = 11 + 11$.

TABLE 2
Summary of Hypothesized Framings in Episode 1c

<i>Framing</i>	<i>Before Reframing</i>	<i>After Reframing</i>
Components in foreground	Expressions written as answers	An answer, $2n + 1$, a process of evaluating that expression by substituting numerical values for n , and values of the function taken to be correct
Relations between components	Different expressions written by Dani and Gillian	Results of the process agree with values of the function taken to be correct

below. Dani argued by evaluating her formula for several values of the step number (i.e., n), and Gillian accepted Dani's formula; they jointly calculated the answer to another question using Dani's formula. We hypothesize that Dani and Gillian's common ground included a schema for evaluating a simple algebraic expression, such as $2n + 1$, by substituting a value for n and carrying out the indicated arithmetic and that Gillian's accepting the formula resulted from her activating that schema to frame Dani's demonstrative explanation. This is summarized in Table 2.

Following Gillian's "Take the what?" (turn 26) Gillian and Dani had the following dialogue:

27. D: Step number. Multiply it by two and add one.
 28. G: (*quietly*) multiply
 5:43 29. D: (*leaned across the desk and pointed to G's homework*) So three times two is six—one, two three, four, five, six—and add one, seven. (*pulled her hand back to her own desk*) And it works for each one.
 30. G: It does?
 31. D: Yeah.
 5:49 32. G: (*writing*) So it would be step number ten, times (*looked up at D*)
 33. D: two is twenty
 34. G: (*writing on her own paper*) twenty
 35. D: and then plus one is twenty-one.
 36. G: Oh, okay.
 5:59 37. D: You can tell it's right by—do number five, times two is one, two, three, yeah
 38. G: n times two, plus one. I didn't see that coming
 6:08 39. D: Oh wai wait. Yeah. Oh, I'm stupid! Function one. I had that right here.
 40. K: (*looked up from her homework, which she had been completing*) So the answer's twenty-one?
 41. D: Yeah, twenty-one.

42. G: (*holding up page 2 of the homework sheet*) Were we supposed to put functions?⁷
- 6:15 43. D: For where? For this one? (*tapped on her own paper*) For the graph?
44. G: Yeah.
- 6:20 45. D: Okay, for the graph, all you do is like—for number one it's three, five
46. G: Yeah, yeah, yeah. I know, I know, I know.
47. D: And then you just write what the thing is . . .
- 6:26 48. G: I got twenty-one, and then what was seventeen (*looked at D's paper*)
Thirty-four? (*looked at her own paper*) Thirty-five, right?
49. D: (*pointed to her own paper*) Yeah, 'cause n times two plus one. Yeah, thirty-five.
- 6:38 50. G: Two n plus one (*finished writing*).

We explain Dani's actions of justifying her formula by hypothesizing a schema of evaluating an algebraic expression by substituting a numerical value for its variable and carrying out calculations indicated by the formula. Although the first five numerical values of the function were associated with diagrams, Dani did not refer to properties of the diagrams in confirming that numerical values obtained by evaluating algebraic formulas agreed with the numerical values that had been determined. We hypothesize that Dani's semantic interpretation of her formula was that the coefficient referred to the (constant) difference between successive numerical values in the sequence and that the added number referred to the difference between the coefficient (times 1) and the first number of the sequence.

To explain Gillian's actions in accepting Dani's proposed answer we hypothesize that Gillian knew and recognized the schema of substituting a value into a formula and calculating a value of the expression by performing arithmetic operations. Gillian contributed to their joint verification of Dani's formula of Function 1, Diagram 10, at turn 32. She acknowledged at turn 38 that Dani's formula was different from her own, then used that formula in calculating the answer for another value of n , restating the formula in turn 50.

Episode 1d: Achieving Mutual Understanding (Less Firmly) of Gillian's Formula

Dani and Gillian proceeded to work on Function 2. They agreed on an answer to the first question, but they had written different formulas for the second question, analogous to their different formulas for Function 1. They made an effort to reach mutual understanding that was partially successful. We infer that at the end of

⁷On page 2 of the homework assignment students were asked to draw graphs of the three functions and were given tables with blanks to fill in, including the value of each function for Diagram 17, which Gillian and Dani discussed in turns 48–49.

this episode Dani and Gillian still had framings that were partially misaligned, in that Dani's framing still focused on numerical evaluations of her formula and Gillian's framing focused on differences between successive diagrams. They had the following dialogue:

50. G: . . . Ok, now, function two.
- 6:40 51. D: I got forty-one.
52. G: Same here. It goes up by four each time?
53. D: Yeah.
54. G: And my function was—five plus—plus n minus one times four.
- 6:55 55. D: Wait. I got four n plus one.
56. G: Four n plus one?
57. D: Yeah, 'cause of ten, you do ten times four.
58. G: It's the same thing, isn't it?
- 7:02 59. D: Wait. What'd you say again?
60. G: Four, um—I did n minus one times four
61. D: Yeah, same thing
62. G: Plus five
- 7:08 63. D: Plus five?
64. G: Five from the beginning (*tapped the first diagram with her eraser, counting under her breath*) because there's five in the beginning and then it goes up by four (*tapped between the diagrams*)
65. D: Yeah, I guess that should work
66. G: Yeah
67. D: Just like, I guess mine's sort of like more simplified or something.

In turns 51 and 52 Dani and Gillian determined that they had the same answer for the first question, the number for Step 10 (41). Then in turns 52 and 53 they determined that the difference between successive terms was four (perhaps the number 4 or four toothpicks). In turns 54 and 55 they determined that they had written different formulas for the next question about the pattern. In turn 58 Gillian said "It's the same thing, isn't it?" which we find ambiguous,⁸ but at that point in the conversation Gillian could have challenged Dani's explicit statement of Dani's formula, so her comment that it was "the same thing" probably functioned as acceptance.

⁸The ambiguity results from indeterminacy of what "the same thing" was. It is possible that they meant that Dani's and Gillian's formulas were "the same," presumably mathematically equivalent, which they were. However, if this is what they (or either of them) meant it is surprising that there was no reference to formulas being equivalent for Function 3, which we discuss as Episode 1e. We infer then that "the same thing" more likely referred to the similarity between Dani's formulas for Functions 1 and 2. On this latter interpretation Gillian's action can be understood as acceptance of Dani's formula for Function 2 on the same basis as she accepted the answer Gillian constructed for Function 1.

We infer that Gillian accepted Dani's formula as giving the correct answer, but then Gillian reexplained her own formula and Dani expressed uncertainty in accepting that. Our explanatory hypothesis is that Dani's framing continued to focus on numerical values of the function, arrived at by evaluating her expression, whereas Gillian's framing focused on an iterative operation of adding segments (i.e., toothpicks) to each diagram to produce its successor.

Our explanation of this episode uses a hypothesis that Gillian and Dani had different cognitive framings, with Gillian's framing focused primarily on properties of diagrams and Dani's focused primarily on properties of numbers. The two formulas are equivalent; therefore, they cannot be distinguished by different results of applying them. Gillian had acquiesced to Dani's persistent demonstrations that her formula gave correct results (turns 38 and 50), but Gillian did not abandon her commitment to her formula. At the same time Dani's acceptance of Gillian's explanation was uncertain: "I guess that should work" and "I guess mine's sort of like more simplified or something" (turns 65 and 67). We propose that the tentative nature of these responses is explained by the hypothesis that Dani's and Gillian's cognitive framings differed, so that each of them presented an argument that was plausible but did not fit securely into the other's structure of understanding.

To explain Gillian's actions in reasserting her formula we hypothesize that she constructed a semantic interpretation of her formulas, with the coefficient referring to a relation between successive diagrams. For Function 2 her formula was "n minus one times four . . . plus five," and we hypothesize that she interpreted this with "four" referring to the difference between successive diagrams (adding four toothpicks), with "five" referring to the number of toothpicks in the first diagram, and with "n minus one" referring to the number of increments that were performed to reach the n th diagram.

At the end of this episode the evidence of mutual understanding that Dani and Gillian provided each other was tentative and qualified. Gillian did not explicitly reject Dani's formula, but by persisting in stating and explaining her own formula Gillian indirectly maintained it as an alternative. And Dani did not explicitly reject Gillian's formula, but her statements of acceptance were expressed with uncertainty ("I guess"). The explanation that we propose of their respective actions is a hypothesis that Dani and Gillian had different conceptual framings, with Dani's framing focused on properties and relations of numbers detached from properties and relations of diagrams and with Gillian's framing focused on properties and relations of diagrams along with their numerical values. We propose this hypothesized difference in framing as an explanation of the partial, tentative nature of Dani and Gillian's mutual understanding. Each of them had a framing in which the other's statements could be incorporated only partially. Their partial successes in these interpretations were sufficient for them to not challenge each other's assertions explicitly, but their lack of alignment prevented them from reaching firm mutual understanding.

TABLE 3
Summary of Hypothesized Framings in Episode 1e

<i>Framing</i>	<i>Before Reframing</i>	<i>After Reframing</i>
Components in foreground	Numbers and formulas written as answers, and a process of evaluating formulas that supports Dani's previous formula	Same as before reframing
Relations between components	Numerical answers for Diagram 10 and formulas are not the same; Dani's formula is "the same thing" as Dani's previous formulas	The process of evaluating formulas supports both Dani's and Gillian's formulas, and a correct answer, 32, is the value of Gillian's formula for Diagram 10

Episode 1e: Achieving Alignment and Mutual Understanding of Gillian's Formula

For Function 3 Gillian and Dani initially had different numerical answers about Diagram 10. Instead of resolving their answers by iterative addition as they had for Function 1 they discussed the formulas they had written for the next question. Dani stated her formula, which was analogous to her formulas for Functions 1 and 2. Gillian then stated her own formula, which was analogous to her (Gillian's) formulas for Functions 1 and 2 and also different from Dani's formula in the same way as before. After a few turns Dani and Gillian reached agreement that Gillian's formula, as well as Dani's, was acceptable (their term: it "works, too"). We explain the change between their discussions of Function 2 and Function 3 by hypothesizing that their communication about Function 3 was supported by cognitive framings that were better aligned than was the case in their communication about Function 2. Dani argued for her formula by showing that it gave the correct answer for three diagrams, using substitution of values and arithmetic operations, as she had previously. Then Gillian adopted that framing—that is, she substituted values into her own formula and showed that it too resulted in the numerical values that Dani and Gillian had accepted as correct based on Dani's previous demonstration. This interpretation is summarized in Table 3.

They had the following dialogue:

- 7:25 68. D: Uhh . . . Function three. I got thirty-two for ten.
69. G: I got thirty-six.
- 7:34 70. D: I did the pattern is the step number, multiply it by three, plus two. Multiply three, then add two, because you have five, go one, two, three four, . . . Wait, times three.

- 7:45 71. G: Wait, it goes five? And then you get one, two, three? And then one, two, three, one, two, three . . .? So, I thought it was five plus three times n minus one.
- 7:55 72. D: Well, ok. Can I see something here first? (*showed G something on D's paper*) Here's one. Multiply it by three—one, two, three—and then add two and that's five. And then two times three is one, two, three, four, five, six, and then add two. And then three is times three—one, two, three, four, five, six, seven, eight, nine—and then add two. (*looked up at G*) So, like that works for each one each time, if you notice.
- 8:18 73. G: Yeah, this, the other thing works too, cuz five plus, see cuz if you do um, n minus one it's zero, so three times zero is zero
74. D: uh huh
75. G: And you still have the five (*looked up at D*)
76. D: uh huh
77. G: (*looked back down at her homework*) For two, you do um, five—uh, you do n is two, minus one is one, one times three is three plus five. And that works. (*looked up at D*)
- 8:36 78. D: You get eight? (*looking at G's paper*) Yeah, ok, yours works too.
- 8:42 79. G: So it works. I must have done bad calculations on ten, though. I did five plus—so it's ten minus
80. D: one is nine
81. G: nine times three is (*looked at D*)
82. D: twenty-seven
83. G: plus five is
84. D: thirty-two
85. G: It is?
86. D: Yeah.
- 8:58 87. G: That's where I went wrong then (*erased on her homework sheet*) I got thirty-six instead of . . .

We infer that achieving aligned framings involved a shift by Gillian. Gillian attended to Dani's demonstration (turn 72) that Dani's formula gave the correct values for Diagrams 1, 2, and 3. We hypothesize that when Gillian responded (turn 73) she argued for her own formula using the same schema that Dani had used, substituting values into the formula and carrying out arithmetic operations, resulting in the answers they had accepted for the first two diagrams. We hypothesize that in her attention to Dani's demonstration, Gillian positioned herself as a listener and attended to Dani as a source, and in this positional framing Gillian recognized the schema of evaluating a formula to match numerical results. The shift by Gillian to framing the problem using the same schema that Dani had used resulted in a response by Dani that we infer involved her reaching understanding with Gillian that Gillian's formula also was a correct answer. Dani responded (turn 78) by treating the answer 8 for Diagram 2 as informative and persuasive, after which Dani and Gillian jointly used Gillian's formula to calculate a value

for Diagram 10, substituting 10 into Gillian's formula and arriving at the answer that Dani had computed for that diagram, which Gillian accepted as a correction (turn 87).

To conclude: Gillian and Dani achieved mutual understanding in their discussion of formulas for Function 3, which they had not achieved in their discussions of formulas for Functions 1 or 2. To explain their reaching the partially mutual understandings that they achieved for Functions 1 and 2 we hypothesize that their respective conceptual framings of the problem were misaligned.⁹ We hypothesize that Dani's framing was a schema of substituting values into a formula and calculating numerical results, matching those with answers that had been accepted as correct. Gillian's framing in their discussions of Functions 1 and 2 focused on relations between diagrams, particularly the number of toothpicks added to each diagram to form its successor, along with the number of toothpicks in Diagram 1. In Episode 1e, in which Gillian and Dani discussed Function 3, Gillian framed an argument for her formula with the schema of substituting and calculating numerical values. Dani accepted this ("yours works too"), which we explain by the hypothesis that this framing by Gillian was aligned with Dani's framing.

CASE 2: CO-CONSTRUCTING AN ALIGNED FRAMING

In Case 1 we hypothesize a process of reaching mutual understanding in which Gillian, who was positioned as a listener attending to a demonstration by Dani, who was positioned as a source, recognized a schema that provided a cognitive framing aligned with Dani's framing, which Gillian adopted to persuade Dani that Gillian's formula was also correct. In Case 2 we present an example in which we hypothesize that participants achieved mutual understanding without having a known schema sufficient for that accomplishment. Instead, they co-constructed a framing, and we explain their interaction by hypothesizing that they were attuned to constraints that they mutually found ways to satisfy.¹⁰ Case 1 and Case 2 both

⁹We expect that Scherr and Hammer (2009) might characterize the difference between Gillian's and Dani's framings as epistemological, involving the relevance of different kinds of knowledge—knowledge of numerical properties and operations in Dani's framing and knowledge obtained from observing properties of the diagrams, with knowledge of numerical operations secondarily, in Gillian's framing. We also consider it valid to characterize this difference as misaligned conceptual framings that differ in which aspects of the situation were foregrounded. In either interpretation there were significant aspects of epistemological framing for which Dani and Gillian were aligned. For example, they were satisfied to conclude that their equations both "work," and they did not take up the question of whether their equations were mathematically equivalent. We explain this by hypothesizing that their epistemological framing did not extend to issues beyond being convinced that written answers were correct.

¹⁰This case was also discussed by Greeno and van de Sande (2007).

illustrate the importance of careful attention by at least one participant to another participant's framing. In Case 1 Gillian attended to Dani's solutions and recognized a schema that framed those solutions. Gillian then used that framing successfully to advocate for her own formula. In Case 2 Ms. Sanchez attended to information presented by Gillian and recognized that her own understanding was not aligned with Gillian's framing. But Ms. Sanchez did not recognize a schema that she could adopt to achieve alignment with Gillian's framing. Instead, Ms. Sanchez positioned herself as leader of an interaction in which she was a listener, with Gillian positioned as a source, and they, along with another student, jointly constructed a way to solve the problem with a different conceptual framing from the one Ms. Sanchez had initially.

The interaction of this case was a whole-class discussion that occurred late in the school year, in May. The class was studying a unit on quadratic equations. Ms. Sanchez began the lesson by reinstating a problem the class had begun to work on in the previous class session. The text of the problem was the following:

Sally's back yard is 40 feet wide by 72 feet long, and it is structured so that a central rectangle of grass is surrounded by an even border of flowers. The area of the border (flowers) is $\frac{5}{12}$ of the area of the whole garden. If Sally wants to be sure to walk her dog for .25 mile, how many laps will she have to take around the garden?

In the previous session the class had calculated the areas of the inner grass rectangle (1,680 square feet) and the border (1,200 square feet). Ms. Sanchez drew a diagram on the whiteboard with these results and the given total length and width labeled (see Figure 2).

Episode 2a: Reaching an Impasse

Ms. Sanchez added to the diagram shown in Figure 2, drawing two question marks, one in the left vertical border and the other in the top horizontal border. She referred to work done by one of the groups in their previous class session using a guess-and-check strategy.

We hypothesize that Ms. Sanchez planned to solve the problem by defining the width of the border as her variable—call it w —giving the quadratic equation $(72 - 2w)(40 - 2w) = 1680$, which could be solved for w ; then determining the length $(72 - 2w)$ and width $(40 - 2w)$ of the inner rectangle; then finding its perimeter; and finally calculating the number of laps needed to walk .25 mile. Ms. Sanchez was surprised by the students' lack of response, especially having drawn question marks to indicate the two equal quantities that could be referred to by a symbol for a variable.

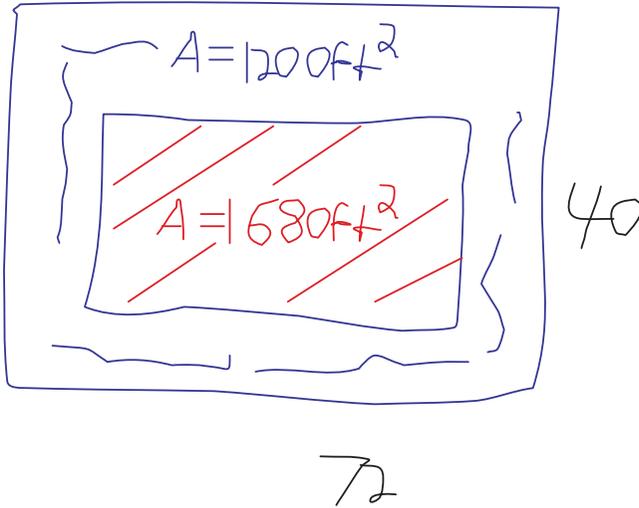


FIGURE 2 Garden lap problem diagram from whiteboard (color figure available online).

7. Ms. S: . . . guess and check. We want to also be able to write equations that will hopefully help us be able to solve problems like this. So, to have an equation, we need to have a variable to write the equation. So, in this problem—I kind of gave it away—what I wrote, oh well, umm, what would my variable be?
- 3:08 *(pause, students were silent)*
- 3:18 8. Ms. S: Oh, not all at once. What would my variable be in this? What am I trying to figure out here? (Greeno & van de Sande, 2007, p. 18)

Episode 2b: Beginning a Proposal That Is Anomalous in Ms. Sanchez's Framing

Ms. Sanchez's plaintive question (turn 8) elicited a hesitant response from Gillian. Ms. Sanchez called on Gillian, who came to the whiteboard and suggested the beginning of a proposal for setting up the problem. Gillian began by defining variable symbols, x and y , that referred to the dimensions of the inner rectangle. These are quantities that need to figure in a solution to the problem, but they were not specified explicitly by labeling them as variables in the plan that we hypothesize Ms. Sanchez had constructed. Instead, we hypothesize, they would be calculated easily after the width of the border had been found. We infer, therefore, that Ms. Sanchez was initially unable to understand Gillian's proposal because its conceptual framing was misaligned with Ms. Sanchez's framing. Their framings contained the same elements, but they had different relations. We summarize this interpretation in Table 4.

TABLE 4
Hypothesized Framings in Episode 2b

<i>Framing</i>	<i>Ms. Sanchez's Framing</i>	<i>Gillian's Framing</i>
Components in foreground	Border width is primary, denoted by “?”	Length and width of inner rectangle are primary, denoted by symbols x and y
Relations between components	Dimensions of inner rectangle can be calculated after border width is found	Border width can be calculated with values of x and y

Ms. Sanchez did not understand how Gillian's proposal could develop toward a solution. We attribute her lack of understanding to a misalignment of framings. The following dialogue between Ms. Sanchez and Gillian occurred, beginning with Ms. Sanchez calling on Gillian:

- 3:29 9. Ms. S: Ok, Gillian?
 10. Gillian (G): You're trying to find out the two lengths of the inside square.
 11. Ms. S: I'm sorry. I couldn't concentrate on what you were saying.
 12. G: You're trying to find out the two lengths of the inside—or the—
 13. Ms. S: The two lengths? What do you mean the two lengths?
 14. G: Of the inside square, like (*pointed toward the board*)
 15. Ms. S: (*pointed to the length and width of the inner rectangle on the board*)
 This?
 16. G: Yeah. You could put them as x and y .
 17. Ms. S: (*labeled the vertical length x and the horizontal length y*) x and y .
 Ok. So, is there any way I can write an equation for this? (Greeno & van de Sande, 2007, p. 18)

The solution that Ms. Sanchez had planned, according to our hypothesis, had the length and width of the inner rectangle in the background as numerical quantities to be calculated using the width of the border after that width was found. Gillian's proposal put the length and width of the inner rectangle in the foreground to be represented explicitly at the beginning. Ms. Sanchez positioned herself as a listener with Gillian as a source, and at turn 17 she asked a question that identified a constraint: the need to construct an equation.

Episode 2c: Co-Constructing Representations Toward a Different Framing

In the next 2.5+ min Gillian and Ms. Sanchez constructed algebraic expressions and established that they referred to the border widths, the quantities that Ms. Sanchez had intended to be represented by a single variable.

TABLE 5
Hypothesized Shared Framings Before and After Episode 2c

<i>Framing</i>	<i>Beginning of Episode 2c</i>	<i>End of Episode 2c</i>
Components in foreground	Quantities in the problem situation: length and width of inner rectangle and border widths	Expressions $(72 - y)/2$ and $(40 - x)/2$, along with length and width of inner rectangle and border widths
Relations between components	Length and width of inner rectangle are primary, denoted by “x” and “y,” and equations are needed	Expressions refer to vertical and horizontal border widths, respectively, which are denoted by “?” in Ms. Sanchez’s initial framing; equations are still needed

We hypothesize that at the beginning of Episode 2c Ms. Sanchez adopted a conceptual framing from Gillian’s presentation, shifting the perspective of her framing by placing the length and width of the inner rectangle and the symbols x and y in the foreground, but she also had expressed the need for an equation. By the end of Episode 2c Gillian had written expressions with x and y that referred to the vertical and horizontal border widths. These new components of the situation were not sufficient to solve the problem, but with them, the framing that Gillian and Ms. Sanchez had constructed included connections between quantities and the symbols that referred to them that were not there at the beginning of the episode. This interpretation is summarized in Table 5.

Gillian and Ms. Sanchez, with a presentation by Hannah, had the following conversation:

18. G: Umm—I came up with—umm— x divided by—or forty minus x divided by two.
19. Ms. S: Forty—minus x —divided by two. Where’d you get that from?
20. G: Can I go up and show you $\langle xxx \rangle$? (*went to the board*)
21. Ms. S: (*walked toward back of classroom*)
- 4:30 21. G: This length is forty. (*pointed to vertical length of the outer rectangle*) And this is x . (*another vertical gesture*) So if you do forty minus x divided by two, you’ll get the two (*pointed to the two vertical borders*) widths, I think. That’s what I came up with. I don’t think it’s right, I don’t know if it’s right. And then seventy-two minus y (*pointed to “y” on the diagram*) divided by two will get those (*pointed to the horizontal borders*) I don’t know. I tried. (*pause*) Does it work?
22. Unidentified student: You could make it into, like—
23. G: ‘Cause $\langle xxx \rangle$ (*pointed to parts of the diagram*)
24. Ms. S: Could you write out what you’re saying? I’m having a hard time hearing it.

25. G: (*wrote "72 - y"*) So, if you do seventy-two minus y
26. Ms. S: Uh huh
27. G: (*wrote "40 - x"*) and like forty minus x. And then you'll get the—and then divide—'cause you're trying to get this here, right? (*pointed to the top and bottom horizontal borders*) So then you'd divide it by two.
28. Ms. S: You're trying to get what?
29. G: 'Cause remember how you said that we were trying to get what this is (*pointed to the top horizontal border in the diagram*)
30. Ms. S: Uh huh
31. G: So if you divide it (*pointed to "72 - y"*) by two you should get this and this (*pointed to top border's width*)
- 5:30 32. Ms. S: Yeah, but I don't have enough numbers to get it though, do I?"
33. G: (*wrote a fraction line and "2" below "72 - y" and below "40 - x"*)
34. Hannah (H): Well, you do if you know what x and y are.
35. G: Yeah, that's what I'm trying to say, 'cause you said that we need a variable and so—
36. Ms. S: Ok, and so then how would I write an equation with those things?
37. G: Umm, I don't know.
38. Ms. S: Does anybody have—
39. G: I wrote this yesterday. I just—
40. Ms. S: Ok. No. So, do you guys understand what—can you point out where the seventy-two minus y is going to be that question mark?
41. G: (*walked toward her seat*)
- 6:00 42. Ms. S: One of the question marks? Which question mark is it going to be?
43. G: (*walked back to the board*) Oh it would be the, this one (*pointed to the right vertical border*)
44. Ms. S: Ok.

What Gillian and Ms. Sanchez accomplished in this excerpt was mutual understanding that the expressions $(72 - y)/2$ and $(40 - x)/2$, which Gillian wrote on the board, referred respectively to the widths of the vertical and horizontal borders. Ms. Sanchez was positioned as a listener, and Gillian was the main source in their joint inquiry. Gillian was already positioned as a source from proposing the labeling of x and y as the dimensions of the inner rectangle.

This involved a different conceptual framing from the one Ms. Sanchez had tried to introduce. In Ms. Sanchez's initial framing the border width was in the foreground as a variable to be represented in a quadratic equation, and when that was solved the resulting value would support calculating the length and width of the internal rectangle. That is, calculating the length and width of the inner rectangle would depend on first determining the numerical value of the border width. The expressions that Gillian presented reversed the relation between the border widths and the inner rectangle dimensions. In Gillian's expressions the length and width of the inner rectangle were represented explicitly, and if they were known, their values could be used to calculate the widths of the borders. In contrast, in

Ms. Sanchez's plan the width of the border was primary. Gillian framed her presentations as being relevant to Ms. Sanchez's initial proposals ("Cause remember how you said that we were trying to get what this is," turn 29). Gillian persisted in explaining that her expressions referred to the border widths, and Ms. Sanchez accepted that claim in turn 44.

Although Ms. Sanchez accepted that Gillian's expressions referred to relevant quantities in the problem, she also presented questions about whether they could be used in solving the problem. At turn 32 she asked ". . . but I don't have enough numbers to get it though, do I?" and at turn 36 she asked ". . . so then how would I write an equation with those things?" We characterize these questions as presentations of constraints of solving problems algebraically that Ms. Sanchez took as not being satisfied when she asked about them.

Episode 2d: Co-Constructing a Coherent Conceptual Framing

In this episode Gillian and Ms. Sanchez, with significant participation by another student, Hannah, constructed equations that could be solved to find the dimensions of the inner rectangle. These equations were new components that were added to the conceptual framing that Gillian and Ms. Sanchez had during Episode 2c. The area of the inner rectangle was a quantity that was brought back into the foreground as the referent of one of the equations. The framing that Ms. Sanchez, Gillian, and Hannah constructed by the end of this episode is summarized in Table 6.

Episode 2d included the following discourse:

45. H: Couldn't you use substitution since you have two equations? I mean, you could make another equation x times y equals 1680?
 46. G: Yeah, I had that last <night>. I just didn't write that down. (*Wrote on the board: $x y = 1680$*) So you have x times y equals

TABLE 6
Hypothesized Shared Framing at the End of Episode 2d

<i>Framing</i>	<i>End of Episode 2d</i>
Components in foreground	Two equations with x and y , and quantities: length, width, and area of inner rectangle and width of borders
Relations between components	Symbols and expressions refer to quantities: length and width of inner rectangle, area of inner rectangle, and border widths; equations are solvable with substitution to find values of the length and width of the inner rectangle

47. H: So, now you have two—two equations and two variables. So, you could, like, do substitution for the other one? And then you can—
48. Ms. S: Ok, so I see one equation—do you guys agree with what she has right there? x times y equals one thousand, six hundred and eighty
49. Other students: Uhh. Well—
50. Ms. S: Do you have a question—any questions for her—for what she's written so far? 'Cause you—I want you to understand what she has there.
51. Norman (N): (*raised hand*)
- 6:53 52. Ms. S: Yeah, Norman?
53. N: Why do you divide by two?
54. G: 'Cause you're trying to figure out this (*pointed to the border in the diagram*)
55. N: Oh, yeah.
56. G: I think it's because you're trying to figure out this and this (*pointed to the diagram*). I, I, I wrote this yesterday, and you just asked, so I thought I'd show and see if it was right.
57. H: Yeah, that is. It's—
58. G: Yeah, so, and I also put this was equal to each other (*pointed to*“(72 - y)/2” and “(40 - x)/2”) but I don't think that's right, so—Dani?
59. Dani (D): Umm. Is seventy-two minus—like seventy-two minus y (*moved her hand horizontally*) Is that <xxx> the length of y or x times y ?
60. G: Yeah, cause it's seventy—it's seventy-two minus y 'cause you're trying to find
61. D: Find
62. G: —these two (*pointed to the vertical left and right borders*) right here. So if you get seventy-two minus y , you're going to get the (*pointed to the vertical left and right borders*).
63. H: And they're both going to equal the same number.
64. G: Yeah. The reason you divide it by two is so that you can find the two separate sides (*held up her hands, forming vertical boundaries; capped the marker and stepped back*) Yeah.
65. Ms. S: So, you said that they're going to equal the same number? How could you—Do you guys agree that those are going to equal the same—what—what's going to equal the same number? x and y are going to equal the same number?
66. Another student: No—the question marks.
67. G: Yeah.
68. H: Seventy-two—seventy-two minus y divided by two and forty minus x divided by two. They're both going to equal the same number.
- 8:10 69. Ms. S: Ok. (*pointed to board*) Can you show that up there? Can you change what's written up there so that you know—so that it shows that those two are equal?
70. H: Like, make another variable?
71. Ms. S: Do we need to have another variable? (*pause*) I think we've got plenty—two variables is—more than enough.
72. H: Well, all I'm saying is that—

73. Ms. S: (*got up from sitting and walked toward the board*) So, she's saying that—Ok—yeah
74. G: They should be equal to each other, right?
75. Ms. S: What should be equal to each other?
76. H: Oh, I get it.
77. G: The seventy-two minus y divided by two and the forty minus x —
78. Ms. S: (*wrote “=” on the board between the two expressions*) <xxx> an equal sign right here.
79. G: Yeah, I put one there. I just didn't know if it was right.
80. Ms. S: (*turned to the class and pointed at the board*) Do you guys agree with that?
81. H: Yeah, Uh-huh.
82. Another student: Yeah.
83. Ms. S: Ok. This is very interesting. This is a completely different way than I looked at it. I only had one (*held up one finger*) variable when I wrote this, so this very—this is hard for me to think about. (*moved to the side of the board*) So you guys are going to get—so that you should get the same answer, cause I don't see—I mean, do you guys see anything wrong with what's going on here? These two equations? (*pause*) You guys agree with everything that's written there?
84. H: Yeah.

A few turns later Ms. Sanchez said, “I started with the question mark. I did it completely different than you guys did. I started with the question mark and my question mark was my variable. But, for you guys—you guys did it this way (*pointed to sides of the inner rectangle*) and I don't see why that wouldn't work as well.”

Ms. Sanchez's positioning continued as a listener, and Gillian was the main source in their joint inquiry. Hannah also positioned herself as a source, volunteering information that she generated that she (correctly) understood provided satisfaction of constraints that Ms. Sanchez had questioned. We interpret Ms. Sanchez's statement “I don't see why that wouldn't work as well” as positioning her as a facilitator, not as a definitive authority.

Hannah entered the conversation as a source: She presented an equation in x and y ($xy = 1680$; turn 45), which helped. Hannah asserted that the two expressions were “both going to equal the same number” (turn 63), which was confirmed and clarified by Gillian, Ms. Sanchez, another student, and Hannah in turns 64–68. In turns 69–78 Ms. Sanchez led an exchange with Hannah and Gillian about constructing an equation based on that equality (the “same number” that the two expressions were equal to did not need to be given a symbol), and Ms. Sanchez added the equals sign that formed the second equation. Ms. Sanchez authorized

the representations as a correct setting up of the problem and emphasized that this was “a completely different way than I looked at it.”

We explain this episode, as well as Episode 1d, by hypothesizing that participants reached mutual understanding by constructing aligned conceptual framings, thereby overcoming a difficulty of understanding caused by misalignment. Case 1 and Case 2 differ, however, in a way that is theoretically significant. In Episode 1d achievement of mutual understanding can be explained by hypothesizing that Gillian recognized a schema of evaluating an algebraic expression and checking the results against known values. This schema was framing Dani’s arguments in support of Dani’s formula, and Gillian’s argument in support of her own formula was framed with the same schema, thus achieving aligned framings. In Episodes 2c and 2d the framing that Ms. Sanchez, Gillian, and Hannah constructed did not occur simply. Instead, components of the problem situation were presented, and a coherent framing gradually emerged. We propose that this example is evidence that a process more general than schema recognition has to be hypothesized; we propose that this process is constraint satisfaction.

Discussion

Our analysis agrees with Ms. Sanchez’s retrospective explanation. We agree that it was difficult for her to understand Gillian’s proposed way to set up the problem (“this is hard for me to think about”), and we agree with her hypothesis that the difficulty was caused by interference with the solution plan that she had constructed (“I did it completely different . . . my question mark was my variable”).

What we add to Ms. Sanchez’s account is the hypothesis that Ms. Sanchez’s difficulty was an example of misalignment of conceptual framings. The information that Gillian presented did not fit with the framing that Ms. Sanchez had initially, so to reach mutual understanding Ms. Sanchez, Gillian, and Hannah adopted a positional framing with Ms. Sanchez as a listener and with Gillian and then Hannah as sources. They constructed a framing in their common ground in which the elements of the problem were organized differently, a process known classically as *restructuring* or *recentering* and studied extensively by Gestalt psychologists (e.g., Duncker, 1935/1945; Wertheimer, 1945/1959). In our account the critical move was made by Ms. Sanchez in Episode 2b, when she focused attention on the quantities that Gillian had mentioned explicitly. Putting the length and width of the internal rectangle in the foreground was aligned with Gillian’s perspective and was the beginning of a framing that Gillian, Ms. Sanchez, and Hannah constructed in their subsequent interaction.

CASE 3: REFRAMING AND REFINING CAUSAL RELATIONS

Case 3 is a reanalysis of excerpts of an episode of interaction recorded and previously analyzed by Roschelle (1992). Two students worked collaboratively on tasks that used an interactive computer program, the Envisioning Machine. The program displayed two windows, called the Newtonian World and the Observable World. The program displayed animations involving the motion of a dot (called a *ball*) in the Observable World and a circle (called a *particle*) with two arrows attached in the Newtonian World that left a dotted-line trace of each motion. Each problem, which Roschelle called a *challenge*, presented a motion of the ball, determined by the program. The program was written so that the motion of the particle was determined by the lengths and directions of the arrows. One arrow, called the *thin arrow*, extended from the center of the particle and represented the initial velocity of the particle's motion. The other arrow, called the *thick arrow*, extended from the head of the thin arrow and represented acceleration. The display is illustrated in Figure 3, where acceleration occurs throughout the motion event.

In each challenge that Roschelle (1992) discussed the students' goal was to adjust the thick arrow so that the motion of the particle would match the motion of the ball. In the tasks analyzed by Roschelle and also by us, the initial position and initial direction and length of the thin arrow (velocity) were given, and the students were to match the motion of the ball by adjusting the thick arrow (acceleration). The computer display also included a window in which the thin and thick arrows were shown separated from the particle. Figure 4 shows examples of this display for an acceleration that occurred in a 1-s burst. The magnitude and direction of the thick arrow represented in the separated arrows display as a movement of the head of the thin arrow from the tail to the head of the thick arrow. After that

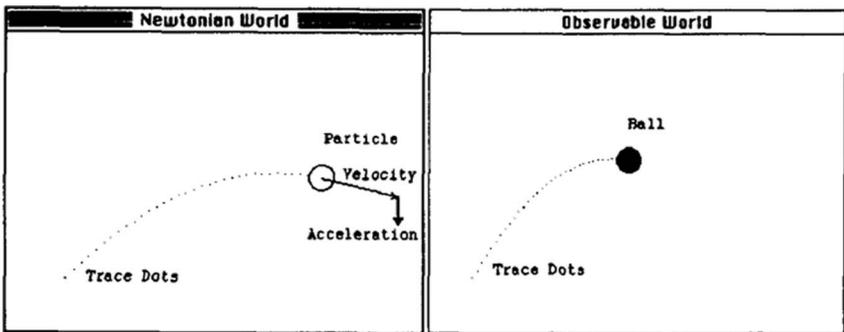


FIGURE 3 The Envisioning Machine. Adapted from Roschelle (1992, p. 340)

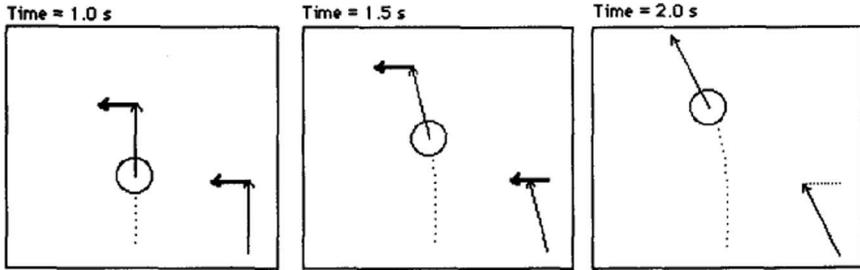


FIGURE 4 Velocity space display (lower right of screen) at three time instances (Roschelle, 1992, p. 240).

happened, a dotted-line trace of the movement of the thin arrow's head remained in the separated-arrows display.

Episode 3a: Co-Constructing a Reframing

Carol and Dana were working in their second session with the Envisioning Machine. In the second challenge in this session the ball moved horizontally to the right, then veered 45 degrees downward, and then continued. The desired trajectory is shown in Figure 5, and the correct solution is shown in Figure 6.

During their work on this challenge Carol and Dana gained a significant insight into the way in which the simulation worked. We interpret their learning as an example of conceptual reframing, and we summarize their achievement in Table 7.

During their work on this problem Carol and Dana had constructed a series of settings for the thick arrow, shown in Figure 7. As Roschelle (1992) noted, the thick arrow settings pointed roughly in the direction that the particle was supposed to travel after its path was bent. When Carol and Dana tested these settings the



FIGURE 5 The desired trajectory, a 45-degree bend (Roschelle, 1992, p. 245).

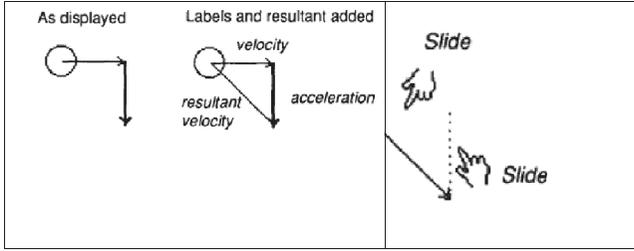


FIGURE 6 The correct solution (Roschelle, 1992, pp. 244–245).

TABLE 7
Hypothesized Framings of the Envisioning Machine in Episode 3a

<i>Framing</i>	<i>Before Reframing</i>	<i>After Reframing</i>
Components in foreground	Thick arrow and direction of motion after change in direction	Thick arrow, thin arrow, and direction of motion after change in direction
Relations between components	Thick arrow's direction causes the direction of the second segment of motion	Thick arrow causes the change of direction of the thin arrow, and thin arrow causes the direction of motion

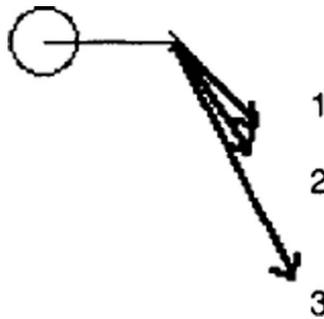


FIGURE 7 Three solution attempts (Roschelle, 1992, p. 246).

particle's path turned in the intended direction but not as sharply as it needed to or as Carol and Dana expected it to. They modified the setting by making the thick arrow longer and its angle closer to vertically downward. Roschelle inferred, and we agree, that in Carol and Dana's understanding the direction of the thin arrow represented the direction of the particle's motion during the first time segment, and the direction of the thick arrow represented the direction of the particle's motion during the second time segment.

Eventually Carol and Dana tried pointing the thick arrow vertically downward, which produced the correct solution. Dana said, “But it doesn’t go at a ninety degree angle. I don’t understand.” Then they continued with the following dialogue:

1. Dana (D): But what I don’t understand is how the lengthening the positioning of arrow—
2. Carol (C): Ooh, you know what I think it is? It’s like the line. Fat arrow is the line of where it pulls that down. Like see how that makes this dotted line. That was the black arrow. It pulls it.
3. D: You’re saying this [dotted line] is the black arrow?
4. C: Yeah.
5. D: and it pulls it the other other arrow (*points to vel [thin arrow] with mouse cursor*) like—
6. C: Like on its hinge. It pulls the other arrow on the hinge down to the tip of the black arrow.
7. Making the line that you see here (*gestures to the trajectory after the 45 degree bend*).
8. C: Right. (Roschelle, 1992, p. 247)

In line 2 Carol foregrounded a relation between the arrows, focusing on a relation between the position of the thick arrow and the change in the thin arrow: “it pulls that down.” The display of a dotted line that traced the path of the thin arrowhead (corresponding to the formerly displayed thick arrow) supported Dana’s understanding, which Dana and Carol confirmed, with the added support of Carol’s reference to “its hinge.” Like Roschelle (1992) we interpret this shift as an example of conceptual change, which we characterize as reframing. The computer program and the tasks for students were designed so that students needed to recognize that the thick arrow caused the thin arrow to change, which in turn caused the motion of the particle.

We take Carol and Dana’s unsuccessful settings of the thick arrow (in Figure 7) as evidence that their initial framing was misaligned with the program’s intended framing.¹¹ They jointly adopted a positioning as a listener, with the computer program as the source. Their shift in conceptual framing included a change in

¹¹The concept of framing is central in some discussions of comprehension of literary texts (e.g., MacLachlan & Reid, 1994), which hypothesize that there is one or more intended framing(s) of a text that make(s) it comprehensible, and successful comprehension of the text involves a reader interacting with the text in a framing or framings that are sufficiently aligned with one or more of the intended framings. Our interpretation of Carol and Dana’s learning is analogous to these discussions of literary interpretation. After they reframed their understanding of the program their framing was aligned with Roschelle’s (1992) intended framing regarding the mediating role of the thin arrow in the relation between the thick arrow and the particle’s motion.

perspective in the sense that it included changing the properties and relations that were in the foreground of their understanding. Before the shift, as we summarize in Table 7, the foregrounded properties were the direction of the thick arrow and the direction of the particle's motion after the path turned; Carol and Dana believed that the direction of this motion should be the same as the direction of the thick arrow. After they reframed their understanding the foregrounded properties included the thick arrow, the direction of the thin arrow, the change of the thin arrow during the turning event, and the correspondence between the thick arrow and the path of the thin arrowhead when the particle's path was turning.

In terms of positional framing Carol and Dana jointly were positioned as a listener, with the computer program positioned as the source. Like Ms. Sanchez in Case 2, whose framing was misaligned with Gillian's framing, Carol and Dana had a conceptual framing that was misaligned with a framing of their source that would support their understanding, and they detected that misalignment when information produced by the program was discrepant from their expectations. Their success in reaching a conceptual framing aligned with the program resulted from their interacting with the program in an exploratory way. We interpret their success as a reframing that was achieved when Carol noticed the correspondence between the thick arrow and the path of the head of the thin arrow, as shown in Figure 5.

Our interpretation of this episode agrees with Roschelle's (1992) in many respects, but we propose characterizing the interaction using rather different general cognitive concepts. Roschelle characterized the students' learning as conceptual change regarding the physical concepts of velocity and acceleration. We are less confident than Roschelle was that the students achieved a conceptual advance regarding those physical concepts. We do not find, at least in the transcriptions Roschelle published, evidence that Carol and Dana interpreted the properties of motion in the animation in those physical terms. (Krange and Ludvigsen, 2008, expressed a similar reservation about this.) We understand that because of the way that the animation was designed and programmed, understanding how the arrows interacted to produce motion on the screen was analogous to understanding corresponding aspects of acceleration and velocity, but we limit our explanatory hypotheses to Carol and Dana's understanding of functional properties of the animations, not the physics concepts. That said, our interpretation has much in common with Roschelle's in its account of the contents of the students' discourse and the understanding that we hypothesize to account for it. Our difference from Roschelle is in our use of the concepts of framing and perspective. We propose that our account in terms of framings and perspectives provides explanations that are more specific than the account that Roschelle gave.

In our account the shift in understanding presented by Carol and jointly accepted was a reframing. By this we mean that the components of the situation in Carol and Dana's understanding were rearranged, with different relations

between them than they had previously. Both the students' initial framing and the one they developed included the two arrows and a trajectory of the ball, to be replicated for the particle, that had two evident segments, with the second segment's direction 45 degrees to the right of the first segment's direction. We hypothesize (as did Roschelle, 1992) that in their initial framing the students thought that the two arrows represented the two segments of motion, so the picture of two arrows should approximate the path of the particle's motion. The relation of the representation to the motion, then, was that the direction of the thin arrow was interpreted as referring to the direction of the first segment of motion and the direction of the thick arrow was interpreted as referring to the direction of the second segment of motion.

The framing that the students achieved in this episode of interaction also included the two arrows and motion along a path with two segments. It also included a change in the direction of the thin arrow and the disappearance of the thick arrow after the direction of motion changed. The direction of the thin arrow was still interpreted as referring to the direction of motion in the first segment. But in this framing the change in direction of the thin arrow coincided with the change in direction of the motion between the segments, and the direction of the thin arrow during the second segment was interpreted as referring to the direction of motion in the second segment. In addition, the direction and length of the thick arrow was interpreted as representing the change in direction of the thin arrow, which was understood as rotating ("on its hinge") so that its head moved from the tail to the head of the thick arrow (in the separated-arrows window shown in Figure 4).

Roschelle's (1992) characterization of this cognitive shift was as follows:

Carol's response to Dana (No. 2) therefore marks the first appearance of a new conceptual structure. In the previous conception, acceleration directly pulls the particle during the second part of the motion. In the new conception, acceleration changes the direction of velocity. (p. 246)

Apart from Roschelle's references to acceleration and velocity (which we call the *thick* and *thin arrows*), our interpretation is in agreement. But in conceptualizing the shift as reframing we focus on relations between the elements in the students' understanding that distinguished the two conceptual structures. In characterizing the shift as reframing we distinguish this kind of cognitive change from others in which the way that components of a situation are organized does not change, and some other change occurs, such as adding new components that can be accommodated without changing the previous framing.

Our characterization of the cognitive shift as reframing is consistent with Roschelle's (1992) discussion. Our account introduces explicit labels for aspects

of the shift that were less explicitly represented by Roschelle. Our perspectival account of framing makes explicit relations of foregrounding, and Roschelle's discussion included several instances in which the students' focus of attention was a significant factor. Roschelle characterized Carol's tracing her finger across regions of the separated-arrows display as "creating a focus of attention on these invisible objects" (p. 244). By a gesture of dragging the point of the thin arrow while she said "it pulls it," "Carol brought the three lines to the foreground" (p. 245). Shifts in attentional focus are inherent in changing or developing a framing, and Roschelle's recognition of these instances supports our view that our theoretical account does not contradict Roschelle's; instead, it provides an explanatory schema that makes some of the features of his account more explicit.

Roschelle (1992) characterized this cognitive shift as conceptual change, which he identified as involving a "deep-featured" situation.¹² We do not disagree with this characterization, but the idea of framing may provide a way of identifying what it is that makes a feature deep. An example from Roschelle's discussion of this episode is the thick arrow, which Dana referred to as "this" when the display contained a dotted-line trace in the space that the thick arrow had been in. One aspect of the thick arrow that might make it a deep feature is that its properties function in the framing in a relatively central way. Another possibility is that the relation of this feature to others in the framing changed significantly in the cognitive shift that occurred, so its properties needed to be salient for the students.

Episode 3b: Learning That Does Not Require Reframing

Our previous example (Episode 3a) occurred following two other episodes discussed by Roschelle (1992). In one of these Carol and Dana extended their framing to include understanding that the length of the thin arrow represented speed and that if the thick arrow was collinear with the thin arrow, the thin arrow would become longer during the simulation. After this achievement they worked on a problem in which the motion in the second segment was to be vertically downward, requiring the thin arrow to point straight down.

During Episode 3b Carol and Dana learned something significant about the simulation, but we believe they did not change their framing significantly. Instead, a relation that they had in their framing at the beginning became more specific. The change that we infer is summarized in Table 8.

¹²Roschelle (1992) also included other features of the process of convergent conceptual change achieved in interaction. These were interplay of metaphors; an iterative cycle of displaying, confirming, and repairing situated actions; and application of progressively higher standards of evidence for convergence. We believe that these features are present in the episodes that we analyze; they may be general features of discourse that qualifies as dialogic or exploratory talk as discussed by Mercer (1995, 2008) and others.

TABLE 8
Hypothesized Framings Before and After Episode 3b

<i>Framing</i>	<i>Beginning of Episode 3b</i>	<i>End of Episode 3b</i>
Relations between components	Thick arrow causes the change of direction of the thin arrow, and thin arrow causes the direction of motion	Thin arrowhead moves from the tail to the head of the thick arrow, and thin arrow causes the direction of motion

We infer that Carol and Dana's framing throughout this episode had the same components in the foreground as it had at the end of Episode 3a, that is the thick arrow, the thin arrow, and the direction of motion in the second segment of the event (see Table 7). The relations at the beginning of Episode 3b were the same as at the end of Episode 3a (see Table 7).

Carol and Dana had the following dialogue:

35. C: So if we wanted to pull this (*vel [head of thin arrow]*) down to there (*a vertical line*). We'd have to have this (*acc [thick arrow]*) all the way around or something like that (*begins to set acceleration [thick arrow], but doesn't release mouse button*).
36. D: No, 'cause that wouldn't that make this (*vel [thin arrow]*) tip swing around to that (*acc [thick arrow]*) tip and make that angle? [*ambiguous gesture*].
37. C: What angle? (*aborts setting acceleration [thick arrow]*)
38. D: So I'm saying, OK
39. C: I bet if I leave it (*acc [thick arrow]*) like that (*releases acceleration [thick arrow]*) as in Figure 8) it's going to make this (*with the mouse, traced the path from center of particle to point of thick arrow*) angle
40. D: Right, that's what I'm saying.
41. C: So we're going to have to swing all the way down here.
42. D: Oh my God! It's all so much clearer now. (Roschelle, 1992, pp. 254–255)
(*Some dialogue between 42 and 43 was omitted. The topic was running the simulation.*)

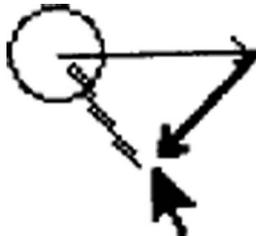


FIGURE 8 The situation constructed for repairing meaning (Roschelle, 1992, p. 256).

43. C: Yeah, see that's right.
 44. D: Oh, that's perfect.
 45. C: It does, it [*thin arrow*] travels right along that edge [*thick arrow*]. So we want it to travel that edge until there [*sets thick arrow*]. 'Cause that will make it [*thin arrow*] come down straight. See it [*thin arrow*] will travel along that edge
 =
 46. D: =Yeah=
 47. C: =Until it's straight down=
 48. D: =So, but what we didn't realize before.
 49. C: Might have to make it a little shorter though. (Roschelle, 1992, pp. 257–258)

We infer that the cognitive change in this episode was that Carol and Dana's understanding of the relation between the thick arrow and the change in the thin arrow became more definite and precise. This was a significant advance in their understanding, to be sure, but it did not involve the kind of rearrangement of relations between components of their situation that we refer to as *reframing*. We infer that Carol and Dana had framings that were quite well aligned with each other and with the intended framing of the computer program, which supported their working out an important detail of the way the simulation worked.

Carol's presentation in line 35 was specific regarding the causal relation between the arrows ("pull this [*head of thin arrow*] down to there"; shown in Figure 6) but imprecise about how much rotation would be needed ("all the way around or something like that"). The rotation she produced on the screen (without releasing the mouse button) would have created a sharper bend in the trajectory than was called for, which Dana apparently recognized (line 36, "No, 'cause that wouldn't that make this [*thin arrow*] tip swing around to that [*thick arrow*] tip and make that angle?") Carol's immediate response was a question (line 37, "What angle?"), and Dana began a clarifying presentation (line 38, "So I'm saying, OK"). However, Carol interrupted, perhaps recognizing the significance of Dana's gestural presentation in line 36, which connected the positions of the arrowheads, and in line 39 Carol presented a different location of the thick arrow and hypothesized its effect on the thin arrow (see Figure 9). Dana's response was confirming (line 40, "right, that's what I'm saying"). We infer that they reached an understanding of the constraint of the program, which corresponds to changing the thin arrow

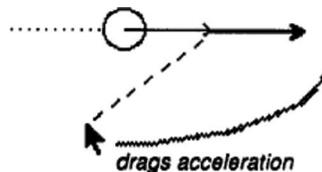


FIGURE 9 Carol's mistaken setting of acceleration (Roschelle, 1992, p. 255).

from its initial length and direction to the resultant of that vector with the vector represented by the thick arrow. They expressed this explicitly in lines 45–48 after we suppose they must have set the thick arrow to point vertically downward to produce a result rotated to about 45 degrees below the horizontal and run a simulation that confirmed their expectation.

In terms of positional framing, in this episode Carol and Dana continued to be positioned jointly as a listener, with the program still positioned as a source. More specifically, however, Carol and Dana each presented a hypothesis that the other evaluated in terms of her own understanding, thus functioning as at least a tentative source for the other's inquiry. Carol stated a hypothesis tentatively (line 35, "or something like that"). Carol's hypothesis focused on the causal relation between arrows that Carol and Dana had learned to foreground in Episode 3a (line 35, "So if we wanted to pull this [*head of thin arrow*] . . . We'd have to have this [*thick arrow*] . . ."). Dana offered an alternative that differed in the specific relation between the orientations of the two arrows. Dana's idea was that the thin arrow would be pulled so that it would extend from the center of the particle to the tip of the thick arrow. Carol's hypothesis was that the thick arrow would have to be rotated so that its tip was beyond the place where the thin arrow should point. This difference was relevant for their understanding of the task, but both Carol and Dana understood that the position of the head of the thick arrow was causally related to the direction that the thin arrow would be in during the second segment of motion. That is, their framings were aligned sufficiently that their difference was meaningful to both of them. (Contrast the difference between Gillian and Ms. Sanchez in Episode 2b, where Ms. Sanchez was unable to understand Gillian's proposal to form expressions with x and y until they had communicated about several aspects of the problem, which we interpret as construction of a framing different from the one Ms. Sanchez had initially.) Carol did not adopt Dana's idea immediately but inquired about her meaning (line 37, "What angle?"). Dana's response (line 38, "So I'm saying, OK") did not answer Carol explicitly, but Carol accepted the idea that Dana had expressed (line 36, "swing around to that [*thick arrow*] tip") and rotated the thick arrow to provide an example of the idea. Dana accepted Carol's statement with strong evidence of mutual understanding, and when Carol applied this idea to the problem they were working on, Dana expressed enthusiasm for the understanding they had achieved (line 42, "It's all so much clearer now").

Our interpretation of this episode agrees with Roschelle's (1992) in many respects but differs in some ways. Roschelle interpreted lines 35 and 36 as a disagreement, leading to a repair cycle. Our interpretation differs in that we interpret the presentation by Carol in line 35 as a kind of question, with the location she put the thick arrow in as a hypothesis, and we interpret the next five turns as constructing an answer to that (somewhat implicit) question rather than resolving a disagreement. This difference between our interpretation and Roschelle's is minor

and does not affect our or his interpretation of the outcome, which was a mutual, more precise, understanding of the way the thick arrow changed the thin arrow in the simulation. As before, we interpret this change more conservatively than Roschelle did, as an understanding of properties of the simulation rather than as an advance that Carol and Dana understood as involving physics principles. We also agree with Roschelle considering as a finding that the participants' understanding, and our interpretation, depended crucially on their use of the material setting and their joint attention to features of the computer display rather than to their speech alone.

Discussion

In their most important respects, Roschelle's (1992) and our accounts of these episodes are in agreement. As we understand his argument, his major goal was to present an account in which "collaboration is analyzed as a process that gradually can lead to convergence of meaning" (p. 235). We share this goal and hope that our analyses (of Roschelle's and our own examples) advance this scientific program. We believe that our account provides a refinement and elaboration of Roschelle's rather than a revision that contradicts explanatory principles that he proposed.

The main dimension of agreement between our account and Roschelle's (1992) is with his concept of convergence, the process in which participants reach mutual understanding. This process is also central in our account. We differentiate reaching mutual understanding at different levels. When participants' framings are misaligned initially, reaching mutual understanding requires that at least one of the participants recognizes the misalignment and either induces (an)other participant(s) to adopt his or her perspective or adopts the perspective of the other participant(s) so that they can communicate with a framing that is shared. This occurred in Episode 3a when Carol recognized the causal relation between the thick arrow and the change in the thin arrow and reframed her understanding, and Dana succeeded in becoming aligned with that reframing. When participants' framings are aligned sufficiently they can add to their understanding by filling in gaps with ideas provided by any of the human participants or, as in the case of Episode 3b here, with recognition of features of the situation that they construct jointly.

In both of the episodes the participants' epistemological framings and general aspects of their positional framing were well aligned. Their accomplishments in reaching new understanding occurred at different levels in the two episodes. In Episode 3a Carol and Donna reorganized their conceptual framing; that is, they constructed a conceptual framing that involved reorganizing (or restructuring or recentering; cf. Duncker, 1935/1945; Wertheimer, 1945/1959) their framing. In Episode 3b the understanding they achieved was a refinement that preserved the framing they had previously.

CONCLUSION

We have presented a theoretical proposal that comprises a collection of assumptions that we have applied in analyses of three cases of problem-solving activity.

Most generally, our analyses contribute toward integrating concepts and methods for understanding cognitive structures and processes with concepts and methods for understanding social interaction by considering informational and interactional aspects of activity as both being integral in understanding and reasoning rather than treating either of these as a context for the other. On the interactional side we develop and evaluate hypotheses about positional framing in activity systems. On the informational side we agree with Hammer and others (Hammer et al., 2005; Scherr & Hammer, 2009) that epistemological framing plays a crucial role in understanding and reasoning. Our focus in this paper is on another level of cognitive framing, *conceptual framing*, which refers to different structures of information, including what is foregrounded (the central focus of attention) and how components of the situation are understood to be related to each other. Historically speaking, issues of cognitive organization were central in Gestalt psychology (e.g., Katona, 1940/1967; Wertheimer, 1945/1959).

In the view that we propose positional and conceptual aspects of framing are integrally related. In order for participants to achieve mutual understanding they need to adopt positional framing in which there is a listener and a source, and information from the source has to be fit into the conceptual framing that the listener has. In our analyses there is significant variability between patterns of interaction in which participants reach mutual understanding. We claim that these various patterns share the important features of positional and cognitive framing that we have identified.

Regarding positional framing, the general feature is the positioning of one or more participants as listener(s) and one or more participants as a source. Our cases show that the source can direct the interaction, with the listener positioned as a recipient (Episodes 1c and 1d); or the listener can direct the interaction, actively questioning the source(s) (Case 2); and the listener function can be performed actively and jointly by more than one participant acting collaboratively (Case 3).

Regarding cognitive framing, in the cases we present here epistemological framing was not problematic for the participants. Participants were generally aligned in the kinds of knowledge they took to be relevant for understanding and solving the problems they worked on. At the same time, in all three cases we analyzed here the participants' epistemological framings were apparently limited to solving specific problems correctly and did not extend to understanding the solutions in relation to general mathematical or physical concepts and principles.

For conceptual reframing to succeed one or more participants detected a misalignment of framings. Mutual understanding was achieved in one case (Case 1) when an inquirer activated a schema that the source had (Episode 1c)

and then adopted that schema to function as a source (Episode 1d). In the other cases (Cases 2 and 3) participants did not know, or at least did not activate, a shared schema. Then a conceptual schema was co-constructed by the participants. We hypothesize that this depended on their having shared knowledge of constraints, which they satisfied in the framing they constructed.

Although we distinguish positional, epistemological, and conceptual aspects of framing analytically, we caution that these are not separately variable factors. For example, in the cases we present here the participants' epistemological framing included the assumption (made tacitly) that knowledge needed to complete the tasks could be gained through inquiry by the participants. But this assumption could not be acted upon unless the participants were positioned in relation to the subject matter as being entitled to conduct inquiry. Another example involves positional and conceptual aspects of inquiry. In Cases 1 and 2 one participant was positioned as the listener (Gillian in Case 1, Ms. Sanchez in Case 2) and (an)other participant(s) were positioned as sources of information that guided the co-construction of a different framing. This positioning determined which participant(s) provided the information that was in the foreground of the framing that the participants constructed.

AN IMPLICATION

The aims of school learning programs include conceptual change in subject matter domains. We conjecture that advances in conceptual understanding, sometimes thought of as theoretical shifts, often involve the development of new resources for framing, as Vosniadou, Vamvakoussi, and Skopeliti (2008) have proposed. An important problem for curriculum and instruction, then, is to support students in developing capabilities for framing information, ideas, questions, and problems—capabilities that they do not bring to the activities in which we hope they will learn.

School learning activities usually position students as needing to understand information, ideas, questions, and problems that are presented by teachers, textbooks, or some other medium. These presentations are framed, and we expect that students often lack resources that are needed to adopt or construct a framing aligned with the framing intended by the teacher or the author(s) of a text or other source that would support good understanding. The idea developed in this paper that understanding requires significant alignment of framings suggests that successful teaching requires support for students to develop framing resources and adopt framings that support understanding the information, concepts, and principles that advance their knowing in a subject matter domain. Examples of learning that accomplishes this have been provided by Brown and Clement (1989), who used a sequence of bridging analogies to extend a student's understanding of

reciprocal physical forces, and by Wagner (2006), who observed a student's extensions of a statistical concept of sampling distributions as a function of sample size. In these examples significant reframing was accomplished by extending the scope of a conceptual resource that was initially known by the student but limited in its application and that later became applicable more generally as the student activated the concept in response to the relevant aspects of a problem or situation.

These examples suggest that a promising pedagogical strategy can recognize students' framings as a critical aspect of the communicative processes of teaching and learning. Research could be conducted to investigate framing resources that support students in making sense of information that they need to process, and to develop an understanding of concepts that are known to be challenging for learners. They could also develop activities in the curriculum that can result in students learning to frame that information and activity productively.

ACKNOWLEDGMENTS

This research was supported by a grant from the Spencer Foundation. We are grateful for extensive conversations with Robert Glaser, who provided comments on previous drafts; with Brian MacWhinney about perspectives in language and reasoning; with David Hammer and Edward Redish about framing; and with Peter Machamer and Jim Bogen about mechanistic explanation. We also thank Jeremy Roschelle, Cindy Hmelo-Silver, Rogers Hall, and Anna Sfard for helpful editorial advice and Ellice Forman, Michael Ford, Robert Hausmann, and Jodi Davenport for a discussion of reviewers' comments on a previous draft.

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